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# The minimum supersymmetric standard model without R-parity through various lepton-number violations

Chao-hsi Chang[1,2], Tai-fu Feng[2]

[1] *CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China**

[2] *Institute of Theoretical Physics, Chinese Academy of Sciences, P.O.Box 2735, Beijing 100080, China*



## Abstract

The minimum supersymmetric standard model (MSSM) without R-parity through various lepton-number violations is investigated systematically. All kinds of possible mixing in the model are formulated precisely. The remarkable issue that the lightest 'Higgs' $H_1^0$ may be heavier than the weak-boson $Z$ at tree level is kept, as the special R-parity violation MSSM where only bilinear violation of the lepton numbers is allowed. It is also shown explicitly that there is a freedom $U(n+1)$ ($n$ is the number of the broken lepton-numbers) in re-defining the lepton and Higgs superfields. Feynman rules relevant to the R-parity violations are given precisely in $'$t Hooft-Feynman gauge. With further assumptions and concerning all available experimental constraints, spectrum for interest sectors is computed numerically.

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*Not post mail address

## I. INTRODUCTION

It is being increasingly realized by those engaged in search for supersymmetry (SUSY) that the principle of R-parity conservation [1,2], assumed to be sacrosanct in the prevalent search strategies, is not in practice inviolable [3]. The R-parity of a particle is defined by $R = (-1)^{2S+3B+L}$ and can be violated either by baryon-number (B) breaking or by lepton-number (L) breaking [3]. Proton decay experiments have set stringent restrictions on the violations of the first and the second generations of the baryon-numbers, but the existent experiment data do not impose so stringent restriction neither on lepton-number violations nor on the third generations of baryon-numbers. In particular, for some cosmology models to explain the baryongenesis, it requests lepton-numbers not to be conserved, that the intensive studies of the supersymmetry models without R-parity, through lepton-number violations and/or the violation for the third generation of baryon-number, have attracted quite a lot of attentions recently[3-22]. As far as the literature is concerned, besides those the so-called basis-independent studies of the R-parity violations [4], the models with lepton-numbers being broken are characterized by certain 'special' Lagrangian which has 'bilinear' [5–9] and/or trilinear [8–12,14] R-parity violations explicitly in superpotential and/or the SUSY soft-breaking terms, and by the violations spontaneously generated by nonzero vacuum expectation values (VEVs) of sneutrinos[3-9]. The supersymmetry models without R-parity can also be arranged well that there will be no contradiction with all the existent experimental data[9-22]. Respect to the very general case, the MSSM without R-parity through various possible lepton-number violations simultaneously has not been investigated thoroughly.

In general, the minimal supersymmetric standard model (MSSM) (R-parity is conserved) has the following general form for the superpotential in terms of superfields:

$$\begin{aligned}\mathcal{W}_{MSSM} = \mu\varepsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + l_I\varepsilon_{ij}\hat{H}_i^1\hat{L}_j^I\hat{R}^I - u_I(\hat{H}_1^2 C^{JI*}\hat{Q}_2^J - \hat{H}_2^2\hat{Q}_1^I)\hat{U}^I \\ -d_I(\hat{H}_1^1\hat{Q}_2^I - \hat{H}_2^1 C^{IJ}\hat{Q}_1^J)\hat{D}^I. \qquad (1)\end{aligned}$$

Here $\hat{H}^1$, $\hat{H}^2$ are Higgs superfields; $\hat{Q}^I$ and $\hat{L}^I$ being quark and lepton superfields (I=1, 2, 3 is the index of generation), all are in doublet of the weak SU(2) respectively. The rest superfields: $\hat{U}^I$ and $\hat{D}^I$ being quark superfields and $\hat{R}^I$ charged lepton ones, but in singlet of the weak SU(2). Here the indices i, j are contracted in a general way for the SU(2) group, and $C^{IJ}$ $(I, J = 1, 2, 3)$ are the elements of the CKM matrix.

When R-breaking interactions are incorporated, the superpotential will be modified as the follows:

$$\mathcal{W} = \mathcal{W}_{MSSM} + \mathcal{W}_L + \mathcal{W}_B \tag{2}$$

with

$$\begin{aligned}
\mathcal{W}_L &= \varepsilon_{ij}[\lambda_{IJK}\hat{L}_i^I\hat{L}_j^J\hat{R}^K + \lambda'_{IJK}\hat{L}_i^I\hat{Q}_j^J\hat{D}^K + \epsilon_I\hat{H}_i^2\hat{L}_j^I] \\
\mathcal{W}_B &= \lambda''_{IJK}\hat{U}^I\hat{D}^J\hat{D}^K.
\end{aligned} \tag{3}$$

Considering the stringent constraint by proton decay experiments on the violations for the first and the second generation baryon-numbers, many authors would like to focus on the third generation baryon-number [8,9,13,18], but the other authors would like to examine the effects of the broken lepton-numbers. Here we will suppress $\mathcal{W}_B$ for all generations totally. The first two terms in $\mathcal{W}_L$ in Eq. (3) have received a lot of consideration, and many restrictions on them have been derived from existing experimental data [4,10–12,20,21]. However, the term $\epsilon_I\varepsilon_{ij}\hat{H}_i^2\hat{L}_j^J$ is also a viable agent for R-parity breaking. It is particularly interesting because it with proper SUSY soft-breaking terms can result in observable factors that cannot be effected by the trilinear terms alone. One of these distinctive effects which we would like to mention here is that, at tree level the lightest neutralino can decay invisibly into three neutrinos, which is not possible if only the trilinear terms in $\mathcal{W}_L$ are presented. In addition, such as non-zero vacuum expectation values (VEVs) of some sneutrinos or/and bilinear violation terms will cause 'fresh' mixing and different phenomenology etc, the interesting results are obtained [5–9]. Whereas what happens to the most general case where all possible lepton-number violations (not only from superpotential but also from the terms

causing SUSY soft breaking) are simultaneously involved, is still an interesting problem to be investigated, thus we would like to turn to the problem in the paper. Indeed in this kind model there is a freedom in principle for defining the superfields, thus remarkable problems, how big of the freedom and how to recognize two different 'parameterization' manners of the same model among this kind of models, emerge. Therefore we will start with the general SUSY version and keep all the possible R-parity violation terms in the superpotential $\mathcal{W}_L$ and in the SUSY soft-breaking Lagrangian properly, then to work out the Lagrangian in 'component version'[1]. All possible mixing and Feynman rules for further precise studies of the phenomenology of the R-parity violation effective theory will be given precisely. The straightforward deductions for these pursers are omitted and lengthy formulas are put into Appendices. Moreover we put the problem aside that the effective theory may have a more fundamental origin at a comparatively high energy scale, although it is interesting and the effective theory may be helpful to trace out some clue on the problem[2]. As for the freedom for defining the superfields, we would also take one section to demonstrate how big it is precisely and make some suggestions on it for later conveniences in later applications.

The paper is organized as follows: in Sect.II, we describe the basic ingredient of the SUSY without R-parity through various lepton-number breaking. The mass matrices of the CP-even Higgs, CP-odd Higgs and charged Higgs are derived precisely. As an important result, relations for CP-even and CP-odd Higgs masses as those in the bilinear case [7], are recovered.

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[1]In fact, we may consider the resultant effective theory as a SUSY superfield version, renormalized at the energy-scale of SUSY breaking. Thus the full renormalization at weak-interaction energy-scale on the parameters in the effective Lagrangian should be made only 'further' in component version.

[2]Some parameters maybe vanish due to higher symmetries if one relates the theory to a specific GUT model [16], whereas the investigation here is still applicable as long as to set the corresponding parameters to vanish.

For completeness, we also give the mixing matrix of charginos and charged leptons, and that of neutralinos and neutrinos. In Sect.III, the Feynman rules for the interactions relevant to R-parity violation, i.e., those of the Higgs bosons (sleptons) with the gauge bosons, and the charginos, neutralinos with gauge bosons or Higgs bosons (sleptons) are presented. The self interactions of the Higgs and the interactions of chargino (neutralino)-squark-quark are also given. In Sect.IV, we examine the freedom [4,20,21] for re-defining the Higgs superfield and the $n-$lepton superfields which are relevant to the R-parity violation. We precisely show the equivalence for two superficially different parameterizations generated by two sets of the $n+1$ superfields, if the two sets of the $n+1$ superfields may be connected by a $U(n+1)$ transformation exactly, hence the $U(n+1)$ transformation can be understood as a freedom for redefining the Higgs and lepton superfields at very beginning. In Sect.V, we try to consider the comparatively interesting 'particle spectrum' numerically under a few further reasonable assumptions on the parameter space of the model partly for simplifying the practical calculations.

## II. THE PHYSICAL MASSES IN THE MSSM WITHOUT R-PARITY

Generally the lepton-number violations in a MSSM not only cause R-parity broken but also make quite a lot of fresh and interesting mixings between particles and sparticles. Let us examine the subject for the model with various lepton-number violations in this section. Since those parts, such as gauge, matter and the gauge-matter interactions etc, in the model are the same as the MSSM, thus we will omit them in the paper everywhere except special needs.

As stated above, we are to consider the superpotential, (to combine Eq.(1) and Eq.(2)):

$$\begin{aligned}
\mathcal{W} = &\mu\varepsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + l_I\varepsilon_{ij}\hat{H}_i^1\hat{L}_j^I\hat{R}^I - u^I(\hat{H}_1^2C^{JI*}\hat{Q}_2^J - \hat{H}_2^2\hat{Q}_1^I)\hat{U}^I \\
&-d^I(\hat{H}_1^1\hat{Q}_2^I - \hat{H}_2^1C^{IJ}\hat{Q}_1^J)\hat{D}^I + \varepsilon_{ij}[\lambda_{IJK}\hat{L}_i^I\hat{L}_j^J\hat{R}^K + \epsilon_I\hat{H}_i^2\hat{L}_j^I] \\
&+\lambda'_{IJK}(\hat{L}_1^I\hat{Q}_2^J\delta_{JK} - \hat{L}_2^IC^{JK}\hat{Q}_1^K)\hat{D}^J \qquad (4)
\end{aligned}$$

with $\mu$, $\epsilon_I$ are the parameters with units of mass, $u^I$, $d^I$ and $l^I$ are the Yukawa couplings as in the MSSM with R-parity, and the parameters $\lambda_{IJK}$, $\lambda'_{IJK}$ describe the trilinear R-parity violation. Since now we consider the case with three families thus the subscripts $I, J, K = 1, 2, 3$. To break the SUSY so as to have a correct phenomenology, the general soft SUSY-breaking terms are introduced accordingly:

$$\begin{aligned}
\mathcal{L}_{soft} = & -m^2_{H^1} H^{1*}_i H^1_i - m^2_{H^2} H^{2*}_i H^2_i - m^2_{L^I} \tilde{L}^{I*}_i \tilde{L}^I_i - m^2_{R^I} \tilde{R}^{I*} \tilde{R}^I \\
& - \sum_I m^2_{HL^I} (H^{1*}_i \tilde{L}^I_i + H^1_i \tilde{L}^{I*}_i) - \sum_{I \neq J} m^2_{L^{IJ}} \tilde{L}^{I*}_i \tilde{L}^J_i - \sum_{I \neq J} m^2_{R^{IJ}} \tilde{R}^{I*} \tilde{R}^J \\
& - m^2_{Q^I} \tilde{Q}^{I*}_i \tilde{Q}^I_i - m^2_{D^I} \tilde{D}^{I*} \tilde{D}^I - m^2_{U^I} \tilde{U}^{I*} \tilde{U}^I + (m_1 \lambda_B \lambda_B \\
& + m_2 \lambda^i_A \lambda^i_A + m_3 \lambda^a_G \lambda^a_G + h.c.) + \{B \varepsilon_{ij} H^1_i H^2_j + B_I \varepsilon_{ij} H^2_i \tilde{L}^I_j \\
& + \varepsilon_{ij} l_{sI} H^1_i \tilde{L}^I_j \tilde{R}^I + d_{sI} (-H^1_1 \tilde{Q}^I_2 + C^{IK} H^1_2 \tilde{Q}^K_1) \tilde{D}^I \\
& + u_{sI} (-C^{KI*} H^2_1 \tilde{Q}^I_2 + H^2_2 \tilde{Q}^I_1) \tilde{U}^I + \varepsilon_{ij} \lambda^S_{IJK} \tilde{L}^I_i \tilde{L}^J_j \tilde{R}^K \\
& + \lambda^{S'}_{IJK} (\tilde{L}^I_1 \tilde{Q}^J_2 \delta^{JK} - \tilde{L}^I_2 C^{JK} \tilde{Q}^J_1) \tilde{D}^K + h.c.\}
\end{aligned} \tag{5}$$

where $m^2_{H^1}$, $m^2_{H^2}$, $m^2_{L^I}$, $m^2_{R^I}$, $m^2_{Q^I}$, $m^2_{D^I}$, $m^2_{U^I}$, $B$ and $B_I$ are the 'bare" mass parameters while $m_3$, $m_2$, $m_1$ denote the masses of $\lambda^a_G$, $\lambda^i_A$ and $\lambda_B$, the $SU(3) \times SU(2) \times U(1)$ gauginos. $d_{sI}$, $u_{sI}$, $l_{sI}$ $(I = 1, 2, 3)$ and $\lambda^S_{IJK}$, $\lambda^{S'}_{IJK}$ are the soft breaking parameters that make necessary mass splitting between the quarks, leptons and their supersymmetric partners. To correspond to the superpotential in Eq. (4), all the possible lepton number violation terms for breaking SUSY softly are involved in Eq. (5).

In general, the scalar potential of the model can be written as

$$\begin{aligned}
V &= \sum_i |\frac{\partial \mathcal{W}}{\partial A_i}|^2 + V_D + V_{soft} \\
&= V_F + V_D + V_{soft}
\end{aligned} \tag{6}$$

where $A_i (i = \cdots)$ denote scalar components, $V_D$ the usual D-terms, $V_{soft}$ just the SUSY soft breaking terms given in Eq. (5). Using the superpotential Eq. (4) and the soft breaking terms Eq. (5), we can write down the scalar potential precisely with the following forms:

$$V_F = |\frac{\partial \mathcal{W}}{\partial H^1}|^2 + |\frac{\partial \mathcal{W}}{\partial H^2}|^2 + |\frac{\partial \mathcal{W}}{\partial \tilde{L}^I}|^2 + |\frac{\partial \mathcal{W}}{\partial \tilde{R}^I}|^2 + |\frac{\partial \mathcal{W}}{\partial \tilde{Q}^I}|^2 + |\frac{\partial \mathcal{W}}{\partial \tilde{U}^I}|^2 + |\frac{\partial \mathcal{W}}{\partial \tilde{D}^I}|^2 \, .$$

(7)

As MSSM but more general, the electroweak symmetry in the model is broken spontaneously if the two Higgs doublets $H^1$, $H^2$, and the sleptons as well acquire nonzero vacuum expectation values (VEVs):

$$H^1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_1^0 + \upsilon_1 + i\phi_1^0) \\ H_2^1 \end{pmatrix} \tag{8}$$

$$H^2 = \begin{pmatrix} H_1^2 \\ \frac{1}{\sqrt{2}}(\chi_2^0 + \upsilon_2 + i\phi_2^0) \end{pmatrix} \tag{9}$$

and

$$\tilde{L}^I = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_{\tilde{\nu}_I}^0 + \upsilon_{\tilde{\nu}_I} + i\phi_{\tilde{\nu}_I}^0) \\ \tilde{L}_2^I \end{pmatrix} \tag{10}$$

where $\tilde{L}^I$ denote the slepton SU(2) doublets, and $I = e,\ \mu,\ \tau$, i.e. the indices of the three families for leptons. From Eqs. (6,8,9,10) it is easy to find the scalar potential includes the linear terms as follows:

$$V_{tadpole} = t_1^0\chi_1^0 + t_2^0\chi_2^0 + t_{\tilde{\nu}_e}^0\chi_{\tilde{\nu}_e}^0 + t_{\tilde{\nu}_\mu}^0\chi_{\tilde{\nu}_\mu}^0 + t_{\tilde{\nu}_\tau}^0\chi_{\tilde{\nu}_\tau}^0 \tag{11}$$

where

$$\begin{aligned}
t_1^0 &= \frac{1}{8}(g^2 + g'^2)\upsilon_1(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + |\mu|^2\upsilon_1 + m_{H^1}^2\upsilon_1 \\
&\quad + \sum_I m_{HL^I}^2\upsilon_{\tilde{\nu}_\tau} - B\upsilon_2 - \sum_I \mu\epsilon_I\upsilon_{\tilde{\nu}_I}, \\
t_2^0 &= -\frac{1}{8}(g^2 + g'^2)\upsilon_2(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + |\mu|^2\upsilon_2 + m_{H^2}^2\upsilon_2 - B\upsilon_1 + \sum_I \epsilon_I^2\upsilon_2 \\
&\quad + \sum_I B_I\upsilon_{\tilde{\nu}_I}, \\
t_{\tilde{\nu}_I}^0 &= \frac{1}{8}(g^2 + g'^2)\upsilon_{\tilde{\nu}_I}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + m_{L^I}^2\upsilon_{\tilde{\nu}_I} + \epsilon_I\sum_J \epsilon_J\upsilon_{\tilde{\nu}_J} \\
&\quad - \mu\epsilon_I\upsilon_1 + B_I\upsilon_2 + m_{HL^I}^2\upsilon_1 + \sum_{J\neq I} m_{L^{IJ}}^2\upsilon_{\tilde{\nu}_J}.
\end{aligned} \tag{12}$$

Here $t^0_i$ ($i = 1,\, 2,\, \tilde{\nu}_e,\, \tilde{\nu}_\mu,\, \tilde{\nu}_\tau$) are tadpoles at 'tree level', thus the true VEVs of the neutral scalar fields should satisfy the condition $t^0_i = 0$, ($i = 1,\, 2,\, \tilde{\nu}_e,\, \tilde{\nu}_\mu,\, \tilde{\nu}_\tau$), therefore we obtain:

$$\begin{aligned}
m^2_{H^1} &= -\Big(|\mu|^2 - \sum_I \Big(\epsilon_I \mu - m^2_{HL^I}\Big)\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} - B\frac{\upsilon_2}{\upsilon_1} + \frac{1}{8}\Big(g^2 + g'^2\Big)\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\Big)\Big), \\
m^2_{H^2} &= -\Big(|\mu|^2 + \sum_I \epsilon_I^2 + \sum_I B_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2} - B\frac{\upsilon_1}{\upsilon_2} - \frac{1}{8}\Big(g^2 + g'^2\Big)\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\Big)\Big), \\
m^2_{L^I} &= -\Big(\frac{1}{8}\Big(g^2 + g'^2\Big)\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\Big) + \epsilon_I \sum_J \epsilon_J \frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}} - \epsilon_I \mu \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}} \\
&\quad + B_I \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}} + m^2_{HL^I}\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}} + \sum_{J\neq I} m^2_{L^{IJ}} \frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}}\Big), \qquad (I = e, \mu, \tau).
\end{aligned} \tag{13}$$

For convenience, later on we will call all of these scalar bosons ($H^1$, $H^2$ and $\tilde{L}^I$) as 'Higgs'. As for the scalar sector, the Higgs-boson mass-matrices squared may be obtained by:

$$\mathcal{M}^2_{ij} = \frac{\partial^2 V}{\partial \phi_i \phi_j}|_{minimum}, \tag{14}$$

here 'minimum' means to evaluate the values at $< H^1_1 >= \frac{\upsilon_1}{\sqrt{2}}$, $< H^2_2 >= \frac{\upsilon_2}{\sqrt{2}}$, $< \tilde{L}^I_1 >= \frac{\upsilon_{\tilde{\nu}_I}}{\sqrt{2}}$ and $< A_i >= 0$ ($A_i$ represent all the other scalar fields). Note that the matrices of the CP-even and the CP-odd scalar bosons both are $5\times5$ as we have the sneutrinos which correspond to three left-handed neutrinos;, whereas the matrix of the charged Higgs is $8 \times 8$ as we have the charged sleptons which correspond to three left-handed and three right-handed charged leptons.

Now let us summarize the results and try to make the matrices diagonal in the following subsections.

### A. The mass matrices for Higgs

The mass terms of the CP-even Higgs from the scalar potential Eq. (6):

$$\mathcal{L}^{even}_m = -\Phi^\dagger_{even}\mathcal{M}^2_{even}\Phi_{even} \tag{15}$$

are obtained with the interaction CP-even Higgs fields $\Phi^T_{even} = (\chi^0_1,\ \chi^0_2,\ \chi^0_{\tilde{\nu}_e},\ \chi^0_{\tilde{\nu}_\mu},\ \chi^0_{\tilde{\nu}_\tau})$ as basis, and the corresponding mass matrix is

$$\mathcal{M}^2_{even} = \begin{pmatrix} r_{11} & -e_{12}-B & e_{13}-\mu\epsilon_1 & e_{14}-\mu\epsilon_2 & e_{15}-\mu\epsilon_3 \\ -e_{12}-B & r_{22} & -e_{23}+B_1 & -e_{24}+B_2 & -e_{25}+B_3 \\ e_{13}-\mu\epsilon_1 & -e_{23}+B_1 & r_{33} & e_{34}+\epsilon_1\epsilon_2 & e_{35}+\epsilon_1\epsilon_3 \\ e_{14}-\mu\epsilon_2 & -e_{24}+B_2 & e_{34}+\epsilon_1\epsilon_2 & r_{44} & e_{45}+\epsilon_2\epsilon_3 \\ e_{15}-\mu\epsilon_3 & -e_{25}+B_3 & e_{35}+\epsilon_1\epsilon_3 & e_{45}+\epsilon_2\epsilon_3 & r_{55} \end{pmatrix} . \tag{16}$$

The parameters appearing in the matrix elements are defined in Appendix A. Note that when obtaining the above mass matrix, the Eq. (13) is used.

The physical CP-even 'Higgs' $H^0_i$ (eigenvalues and corresponding eigenstates) are obtained by means of a standard method to make the matrix Eq. (16) diagonal. Namely we may find a unitary matrix $Z_{even}$:

$$H^0_i = \sum_{j=1}^{5} Z^{ij}_{even}\chi^0_j , \tag{17}$$

where $Z^{ij}_{even}$ $(i,\ j = 1,\ 2,\ 3,\ 4,\ 5)$ are the elements of the matrix that converts the mass matrix Eq. (16) into a diagonal one:

$$\mathcal{L}^{even}_m = -\Phi^\dagger_{even}\cdot\mathcal{M}^2_{even}\cdot\Phi_{even} = -\mathcal{H}^{0\dagger}\cdot\mathcal{M}'^2_{even}\cdot\mathcal{H}^0 ,$$

where

$$\mathcal{M}'^2_{even} = Z_{even}\cdot\mathcal{M}^2_{even}\cdot Z^\dagger_{even} = diag(m^2_{H^0_1}, m^2_{H^0_2}, m^2_{H^0_3}, m^2_{H^0_5}, m^2_{H^0_1}) .$$

Thus $\Phi_{even}$ are the 'interaction fields' and $\mathcal{H}^0$ are the physical fields (the eigenstates of the mass matrix).

As for the CP-odd sector of the Higgs, with the 'interaction' basis $\Phi^T_{odd} = (\phi^0_1,\ \phi^0_2,\ \phi^0_{\tilde\nu_e},\ \phi^0_{\tilde\nu_\mu},\ \phi^0_{\tilde\nu_\tau})$, the mass matrix for the CP-odd 'Higgs' can be written as:

$$\mathcal{M}^2_{odd} = \begin{pmatrix} s_{11} & B & -\mu\epsilon_1+m^2_{HL^1} & -\mu\epsilon_2+m^2_{HL^2} & -\mu\epsilon_3+m^2_{HL^3} \\ B & s_{22} & -B_1 & -B_2 & -B_3 \\ -\mu\epsilon_1+m^2_{HL^1} & -B_1 & s_{33} & \epsilon_1\epsilon_2+m^2_{L^{12}} & \epsilon_1\epsilon_3+m^2_{L^{13}} \\ -\mu\epsilon_2+m^2_{HL^2} & -B_2 & \epsilon_1\epsilon_2+m^2_{L^{12}} & s_{44} & \epsilon_2\epsilon_3+m^2_{L^{23}} \\ -\mu\epsilon_3+m^2_{HL^3} & -B_3 & \epsilon_1\epsilon_3+m^2_{L^{13}} & \epsilon_2\epsilon_3+m^2_{L^{23}} & s_{55} \end{pmatrix} . \tag{18}$$

The parameters appearing in the matrix elements are defined precisely in Appendix A.

To be different from the CP-even sector, it is easy, as Ref. [6] from Eq. (18), to find a neutral Goldstone boson (with zero eigenvalue):

$$\begin{aligned} H_6^0 &= \sum_{i=1}^{5} Z_{odd}^{1i}\phi_i^0 \\ &= \frac{1}{\upsilon}(\upsilon_1\phi_1^0 - \upsilon_2\phi_2^0 + \upsilon_{\tilde{\nu}_e}\phi_{\tilde{\nu}_e}^0 + \upsilon_{\tilde{\nu}_\mu}\phi_{\tilde{\nu}_\mu}^0 + \upsilon_{\tilde{\nu}_\tau}\phi_{\tilde{\nu}_\tau}^0), \end{aligned} \tag{19}$$

which is indispensable for spontaneously breaking the EW gauge symmetry. Here the $\upsilon = \sqrt{\upsilon_1^2 + \upsilon_2^2 + \sum\limits_I \upsilon_{\tilde{\nu}_I}^2}$ and similar to the R-parity conserved MSSM, the mass of $Z$-boson $m_Z = \frac{\sqrt{g^2+g'^2}}{2}\upsilon$ is kept. The other four massive neutral bosons can be written as:

$$H_{5+i}(i=2,3,4,5) = \sum_{j=1}^{5} Z_{odd}^{ij}\phi_j^0 \tag{20}$$

where again $Z_{odd}^{ij}$ $(i,\ j = 1,\ 2,\ 3,\ 4,\ 5)$ is the matrix elements and the matrix converts the interaction fields into the physical ones.

From the eigenvalue equations for CP-even and CP-odd 'Higgs' and the identities of the model, similar to the case of Ref. [7], it is not very difficult to find two independent relations for the eigenvalues as follows:

$$\begin{aligned} \sum_{i=1}^{5} m_{H_i}^2 &= \sum_{i=2}^{5} m_{H_{5+i}}^2 + m_Z^2, \\ \prod_{i=1}^{5} m_{H_i}^2 &= \left[\frac{\upsilon_1^2 - \upsilon_2^2 + \sum\limits_{I=1}^{3}\upsilon_{\tilde{\nu}_I}^2}{\upsilon^2}\right]^2 m_Z^2 \prod_{i=2}^{5} m_{H_{5+i}}^2. \end{aligned} \tag{21}$$

The first relation of Eq. (21) is obtained by relating the traces of the two neutral Higgs mass matrices (CP-even and CP-odd) and the second is relating the determinants of the mass matrices. Note: there is a Goldstone in CP-odd sector, thus to obtain the second relation of Eq. (21), the Goldstone mode must have been taken away already, and it is reason why the multi-product in the r.h.s. of the equation is start from 2 (According to the convention here the number 1 corresponds to the Goldstone). If we introduce the following notations:

$$\upsilon_1 = \upsilon\cos\beta\cos\theta_\upsilon\ ,$$

$$
\begin{aligned}
&\upsilon_2 = \upsilon \sin\beta \ , \\
&\sqrt{\sum_{I=1}^{3} \upsilon_{\nu_I}^2} = \upsilon \cos\beta \sin\theta_\upsilon \ ,
\end{aligned}
\tag{22}
$$

the second relation of Eq. (21) becomes:

$$
\prod_{i=1}^{5} m_{H_i}^2 = \cos^2 2\beta m_Z^2 \prod_{i=2}^{5} m_{H_{5+i}}^2 \ . \tag{23}
$$

The first relation of Eq. (21) was obtained in Ref. [22] firstly. The second relation of Eq. (21) was obtained in Ref. [7] firstly in special case of the bilinear R-parity violating.

The two equations are independent, and restrict the masses of the neutral 'Higgs' bosons substantially. As discussed in Ref. [7], for instance, with Eq. (21), Eq. (23) and simple algebra reduction, the upper limit on the mass of the lightest Higgs at tree level

$$
m_{H_1}^2 \leq m_{H_n}^2 \left( \frac{m_Z^2 \cos^2 2\beta}{m_{H_n}^2} \right)^{\frac{1}{n-1}} \frac{1 - \frac{1}{n-1} \frac{m_Z^2}{m_{H_n}^2}}{1 - \frac{1}{n-1} \left( \frac{m_Z^2 \cos^2 2\beta}{m_{H_n}^2} \right)^{\frac{1}{n-1}}}, \tag{24}
$$

is obtained straightforwardly. Here $n \geq 2$ is the number of the CP-even 'Higgs' (the 'original' Higgs and the sneutrinos), $m_{H_1}$ is the mass of the lightest one among them, whereas $m_{H_n}$ is the heaviest one.

On Eq. (24) two points should be noted:

- When $n = 2$ or $m_{H_1}^2 = \cdots = m_{H_n}^2 = m_{H_{n+2}}^2 = \cdots = m_{H_{n+n}}^2 = m_Z^2$, and $\cos^2 2\beta = 1$, the symbol "=" is established.
- In the case of MSSM with R-parity i.e. n=2, $m_{H_1} = m_Z^2 \cos^2 2\beta \frac{1 - \frac{m_Z^2}{m_{H_2}^2}}{1 - \frac{m_Z^2}{m_{H_2}^2} \cos^2 2\beta} \leq m_Z^2 \cos^2 2\beta$ is recovered.

In present case when $n > 2$, such a strong constraint on the lightest Higgs mass $m_{H_1}$ at tree level as that for the R-parity conserved MSSM [2,25]

$$
m_{H_1}^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2
$$

cannot be obtained.

In the MSSM with R-parity, the radiative corrections make the mass of the lightest Higgs larger than that of tree level when completing one-loop corrections and leading two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ are included [26]. For instance the Ref. [27] by precise loop calculations sets the limit on the lightest Higgs mass: $m_{H_1^0} \leq 132$GeV. In the MSSM without R-parity, as indicated here there is no such a stringent restriction on the lightest Higgs at tree level as R-parity conserved one, hence one can quite be sure that the 'theoretical' bound on the lightest Higgs mass must be loosened a lot (i.e. it can be heavier than the restriction from MSSM with R-parity conservation), especially, when loop corrections are involved . In Section V we will show the indications of Eqs.(21, 23) on the lightest Higgs mass more precisely numerically.

### B. The mass matrix for charged Higgs

With the interaction basis $\Phi_c = (H_2^{1*}, H_1^2, \tilde{L}_2^{1*}, \tilde{L}_2^{2*}, \tilde{L}_2^{3*}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$ [3] and Eq. (6), it is easy to obtain the following mass terms for charged 'Higgs':

$$\mathcal{L}_m^C = -\Phi_c^\dagger \mathcal{M}_c^2 \Phi_c, \tag{25}$$

the symmetric matrix $\mathcal{M}_c^2$ is given as Appendix A.

Making the mass matrix diagonal, a zero mass Goldstone boson state:

$$\begin{aligned} H_1^+ &= \sum_{i=1}^{8} Z_c^{1i} \Phi_c^i \\ &= \frac{1}{\upsilon}(\upsilon_1 H_2^{1*} - \upsilon_2 H_1^2 + \upsilon_{\tilde{\nu}_e} \tilde{L}_2^{1*} + \upsilon_{\tilde{\nu}_\mu} \tilde{L}_2^{2*} + \upsilon_{\tilde{\nu}_\tau} \tilde{L}_2^{3*}) \end{aligned} \tag{26}$$

[3]The model which we are considering here is that there is no right-handed neutrinos at all thus there are neutrinos' SUSY partners corresponding to the lift-handed ones, but as for charged leptons, there are not only left-handed ones but also right-handed ones, thus the numbers of 'charged Higgs' are 8 instead of 5 for the 'neutral Higgs' (more than 3 in three generations of leptons.

is obtained. Together with its charge conjugate state $H_1^-$ are needed to break electroweak symmetry and give $W^{\pm}$ bosons masses. With the transformation matrix $Z_c^{ij}$ (to convert the interaction fields into the physical eigenstates), the other seven physical eigenstates $H_i^+$ ($i = 2, 3, 4, 5, 6, 7, 8$) can be expressed as:

$$H_i^+ = \sum_{j=1}^{8} Z_c^{ij} \Phi_j^c \quad (i, j = 1, \cdots, 8). \tag{27}$$

### C. The mixing of neutralinos and neutrinos:

Due to the lepton number violations in the model, fresh and interesting mixing of neutralinos-neutrinos and charginos-charged leptons may happen. We devote two subsections to outline the mixing and solve them numerically late in Sect.V. The piece of Lagrangian responsible for the mixing of neutralinos and neutrinos is:

$$\begin{aligned} \mathcal{L}_{\chi_i^0}^{mass} = & \{ig\sqrt{2}T^a_{ij}\lambda^a\psi_j A_i^* - \frac{1}{2}\frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j}\psi_i\psi_j + h.c.\} + m_1(\lambda_B\lambda_B + h.c.) + \\ & m_2(\lambda_A^i\lambda_A^i + h.c.) \end{aligned} \tag{28}$$

where $\mathcal{W}$ is given by Eq. (4). $T^a$ are the generators of the SU(2)×U(1) gauge group and $\psi$, $A_i$ stand for generic two-component fermion and scalar fields. Writing down the Eq. (28) explicitly, we obtain:

$$\mathcal{L}_{\chi_i^0}^{mass} = -\frac{1}{2}(\Phi^0)^T \mathcal{M}_N \Phi^0 + h.c. \tag{29}$$

with the interaction basis $(\Phi^0)^T = (-i\lambda_B,\ -i\lambda_A^3,\ \psi_{H^1}^1,\ \psi_{H^2}^2,\ \nu_{e_L},\ \nu_{\mu_L},\ \nu_{\tau_L})$ and

$$\mathcal{M}_N = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} \\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1 & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} \\ -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g\upsilon_1 & 0 & -\frac{1}{2}\mu & 0 & 0 & 0 \\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu & 0 & \frac{1}{2}\epsilon_1 & \frac{1}{2}\epsilon_2 & \frac{1}{2}\epsilon_3 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & 0 & \frac{1}{2}\epsilon_1 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & 0 & \frac{1}{2}\epsilon_2 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} & 0 & \frac{1}{2}\epsilon_3 & 0 & 0 & 0 \end{pmatrix}. \tag{30}$$

The mixing has the formulation:

$$
\begin{aligned}
&-i\lambda_B = Z_N^{1i}\tilde{\chi}_i^0, \quad && -i\lambda_A^3 = Z_N^{2i}\tilde{\chi}_i^0, \quad && \psi_{H^1}^1 = Z_N^{3i}\tilde{\chi}_i^0, \\
&\psi_{H^2}^2 = Z_N^{4i}\tilde{\chi}_i^0, && \nu_{e_L} = Z_N^{5i}\tilde{\chi}_i^0, && \nu_{\mu_L} = Z_N^{6i}\tilde{\chi}_i^0, \\
&\nu_{\tau_L} = Z_N^{7i}\tilde{\chi}_i^0
\end{aligned}
\tag{31}
$$

and the transformation matrix $Z_N$ has the property

$$
Z_N^T \mathcal{M}_N Z_N = diag(m_{\tilde{\kappa}_1^0}, m_{\tilde{\kappa}_2^0}, m_{\tilde{\kappa}_3^0}, m_{\tilde{\kappa}_4^0}, m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}). \tag{32}
$$

For convenience as in Ref. [2], we formulate all the neutral fermions into four component Majorana spinors as follows:

$$
\nu_e = \begin{pmatrix} \tilde{\chi}_5^0 \\ \bar{\tilde{\chi}}_5^0 \end{pmatrix}, \tag{33}
$$

$$
\nu_\mu = \begin{pmatrix} \tilde{\chi}_6^0 \\ \bar{\tilde{\chi}}_6^0 \end{pmatrix}, \tag{34}
$$

$$
\nu_\tau = \begin{pmatrix} \tilde{\chi}_7^0 \\ \bar{\tilde{\chi}}_7^0 \end{pmatrix}, \tag{35}
$$

$$
\kappa_i^0 (i = 1, 2, 3, 4) = \begin{pmatrix} \tilde{\chi}_i^0 \\ \bar{\tilde{\chi}}_i^0 \end{pmatrix}. \tag{36}
$$

It is easy from Eq. (30) to find that only one type of neutrinos obtains mass from the mixing at tree level, as pointed out by Ref. [15] firstly, and we will assume it is $\tau$-neutrino naively. One of the stringent restrictions comes from the bound that the mass of $\tau$-neutrino should be less than 20 MeV [23]. Late on for convenience, we will call the mixtures of neutralinos and neutrinos as 'neutralinos' shortly as long as there is no confusion.

### D. The mixing of charginos and charged leptons

Similar to the mixing of neutralinos and neutrino, charginos mix with the charged leptons and form a set of physical charged fermions: $e^-$, $\mu^-$, $\tau^-$, $\kappa_1^\pm$, $\kappa_2^\pm$. In the interaction basis, $\Psi^{+T} = (-i\lambda^+,\ \tilde{H}_2^1,\ e_R^+,\ \mu_R^+,\ \tau_R^+)$ and $\Psi^{-T} = (-i\lambda^-,\ \tilde{H}_1^2,\ e_L^-,\ \mu_L^-,\ \tau_L^-)$, the charged fermion mass terms of the Lagrangian have a general formulation [17]:

$$\mathcal{L}_{\chi_i^\pm}^{mass} = -\Psi^{-T}\mathcal{M}_C\Psi^+ + h.c. \tag{37}$$

and the mass matrix:

$$\mathcal{M}_C = \begin{pmatrix} 2m_2 & \frac{ev_2}{\sqrt{2}S_W} & 0 & 0 & 0 \\ \frac{ev_1}{\sqrt{2}S_W} & \mu & \frac{l_1 v_{\tilde{\nu}_e}}{\sqrt{2}} & \frac{l_2 v_{\tilde{\nu}_\mu}}{\sqrt{2}} & \frac{l_3 v_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{ev_{\tilde{\nu}_e}}{\sqrt{2}S_W} & -\epsilon_1 & \frac{l_1 v_1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I11}v_{\tilde{\nu}_I} & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I12}v_{\tilde{\nu}_I} & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I13}v_{\tilde{\nu}_I} \\ \frac{ev_{\tilde{\nu}_\mu}}{\sqrt{2}S_W} & -\epsilon_2 & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I21}v_{\tilde{\nu}_I} & \frac{l_2 v_1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I22}v_{\tilde{\nu}_I} & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I23}v_{\tilde{\nu}_I} \\ \frac{ev_{\tilde{\nu}_\tau}}{\sqrt{2}S_W} & -\epsilon_3 & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I31}v_{\tilde{\nu}_I} & \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I32}v_{\tilde{\nu}_I} & \frac{l_3 v_1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sum\limits_I \lambda_{I33}v_{\tilde{\nu}_I} \end{pmatrix}. \tag{38}$$

Here $S_W = \sin\theta_W$ and $\lambda^\pm = \frac{\lambda_A^1 \mp i\lambda_A^2}{\sqrt{2}}$. Generally two mixing matrices $Z_+$ and $Z_-$ can be obtained by making the mass matrix $\mathcal{M}_c$ diagonal in a similar way as in the SM to make the mass matrix of quark diagonal i.e. the product $(Z_+)^T\mathcal{M}_C Z_-$ turns to a diagonal matrix:

$$(Z_+)^T\mathcal{M}_C Z_- = \begin{pmatrix} m_{\kappa_1^-} & 0 & 0 & 0 & 0 \\ 0 & m_{\kappa_2^-} & 0 & 0 & 0 \\ 0 & 0 & m_e & 0 & 0 \\ 0 & 0 & 0 & m_\mu & 0 \\ 0 & 0 & 0 & 0 & m_\tau \end{pmatrix}. \tag{39}$$

We denote the mass eigenstates with $\tilde{\chi}$ as follows:

$$-i\lambda_A^\pm = Z_\pm^{1i}\tilde{\chi}_i^\pm, \quad \psi_{H^2}^1 = Z_+^{2i}\tilde{\chi}_i^+,$$

$$\psi_{H^1}^2 = Z_-^{2i}\tilde{\chi}_i^-, \quad e_L = Z_-^{3i}\tilde{\chi}_i^-,$$

$$e_R = Z_+^{3i}\tilde{\chi}_i^+, \quad \mu_L = Z_-^{4i}\tilde{\chi}_i^-,$$

$$\mu_R = Z_+^{4i}\tilde{\chi}_i^+, \qquad \tau_L = Z_-^{5i}\tilde{\chi}_i^-,$$
$$\tau_R = Z_+^{5i}\tilde{\chi}_i^+. \tag{40}$$

The four-component fermions are defined as:

$$\kappa_i^+(i = 1, 2, 3, 4, 5) = \begin{pmatrix} \tilde{\chi}_i^+ \\ \bar{\tilde{\chi}}_i^- \end{pmatrix}, \tag{41}$$

where $\kappa_1^\pm$, $\kappa_2^\pm$ are the usual charginos and $\kappa_i^\pm$ ($i = 3,\ 4,\ 5$) correspond to $e$, $\mu$ and $\tau$ lepton respectively. For convenience, late on we will call the mixtures of charginos and charged leptons as 'charginos' shortly sometimes.

Due to the trilinear terms with coefficients $\lambda'_{IJK}$ (a lepton superfield couples to two quark superfields) in Eq. (4), the squark mixing is also affected by the lepton number breaking interactions. Since it is not the main subject of this paper, besides being discussed elsewhere [28], we outline the effects in the simplest case in Appendix B briefly.

According to the above analysis, we have achieved the formulations of the mass spectrum of the neutralinos-neutrinos, charginos-charged leptons, neutral Higgs-sneutrinos and charged Higgs-charged sleptons. Since the vertices of the interactions are also important, thus in the next section we will give the Feynman rules of the model, which are new to those of the MSSM with R-parity conserved.

## III. THE FEYNMAN RULES FOR THE R-PARITY VIOLATING INTERACTIONS

We have discussed the various masses of the MSSM with R-parity violation. Now, we are discussing the Feynman rules for the model that are new to those in MSSM with R-parity conserved. For convenience in loop calculations, we work out here the rules only in t′Hooft-Feynman gauge [24], which has the gauge fixed terms:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\Big(\partial^\mu A_\mu^3 + \xi M_Z C_W H_6^0\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu B_\mu - \xi M_Z S_W H_6^0\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu A_\mu^1$$

$$+\frac{i}{\sqrt{2}}\xi M_W\Big(H_1^+ - H_1^-\Big)\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu A^2_\mu + \frac{1}{\sqrt{2}}\xi M_W\Big(H_1^+ + H_1^-\Big)\Big)^2$$

$$= \Big\{ -\frac{1}{2\xi}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi}(\partial^\mu F_\mu)^2 - \frac{1}{\xi}(\partial^\mu W^+_\mu)(\partial^\mu W^-_\mu)\Big\} - \Big\{M_Z H_6^0 \partial^\mu Z_\mu$$

$$+iM_W\Big(H_1^+\partial^\mu W^-_\mu - H_1^-\partial^\mu W^+_\mu\Big)\Big\} - \Big\{\frac{1}{2}\xi M_Z^2 H_6^{0^2} - \xi M_W^2 H_1^+ H_1^-\Big\}, \qquad (42)$$

where $C_W = \cos\theta_W$ and $H_6^0$, $H_1^\pm$ are defined as the above. By inserting the expressions Eq. (42) into Lagrangian, the desired vertices for the Higgs bosons are obtained. We assume the relevant parameters are real, i.e. at this moment only CP being conserved is considered, one may find that $H_1$, $H_2$, $H_3^0$ $H_4^0$, $H_5^0$ are scalars but $H_6^0$, $H_7^0$, $H_8^0$, $H_9^0$, $H_{10}^0$ pseudoscalar.

### A. Feynman rules for Higgs (slepton)- gauge boson interactions

Let us compute the vertices of Higgs (slepton)- gauge bosons in the model precisely. The interaction terms of Higgs bosons and gauge bosons are:

$$\mathcal{L}^1_{int} = -\sum_I(\mathcal{D}_\mu\tilde{L}^{I\dagger}\mathcal{D}^\mu\tilde{L}^I - \mathcal{D}_\mu\tilde{R}^{I*}\mathcal{D}^\mu\tilde{R}^I) - \mathcal{D}_\mu H^{1\dagger}\mathcal{D}^\mu H^1 - \mathcal{D}_\mu H^{2\dagger}\mathcal{D}^\mu H^2$$

$$= \sum_I\Big\{\Big[i\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\partial^\mu\tilde{L}^I + h.c.\Big] - \tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu$$

$$-\frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)\tilde{L}^I + \Big(ig'B_\mu\tilde{R}^{I*}\partial^\mu\tilde{R}^I$$

$$+h.c.\Big) - g'^2\tilde{R}^{I*}\tilde{R}^I B_\mu B^\mu\Big\} + \Big\{H^{1\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\partial^\mu H^1$$

$$+h.c.\Big\} - H^{1\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)H^1$$

$$+\Big\{H^{2\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\partial^\mu H^2 + h.c.\Big\} - H^{2\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu$$

$$-\frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)H^2$$

$$= \mathcal{L}_{SSV} + \mathcal{L}_{SVV} + \mathcal{L}_{SSVV}, \qquad (43)$$

here $\mathcal{L}_{SSV}$, $\mathcal{L}_{SVV}$ and $\mathcal{L}_{SSVV}$, are the relevant interaction terms. For the Feynman rules and convenience in practical applications they are precisely rewritten in the physical bases which were obtained in the previous section. Since the precise formulas of $\mathcal{L}_{SSV}$, $\mathcal{L}_{SVV}$

and $\mathcal{L}_{SSVV}$ are lengthy, so we put them into Appendix C. Instead, let us summarize the relevant Feynman rules in Fig. 1 $\sim$ 4 and emphasize a few features about them. First, the presence of the vertices $Z_\mu H_i H^0_{5+j}$ ($i,\ j=1,\ 2,\ 3,\ 4,\ 5$) and the forbiddance of the vertices $Z_\mu H_i H^0_j$ and $Z_\mu H_{5+i} H^0_{5+j}$ ($i,\ j=1,\ 2,\ 3,\ 4,\ 5$) are due to their CP nature. Second, besides the $W^+_\mu Z^\mu H^-_1$ ($H^-_1$ is just the charged Goldstone boson) interaction, there are not vertices $W^+_\mu Z^\mu H^-_i$ ($i=2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8$) at tree level, that is the same as the MSSM with R-parity being conserved and the general two-Higgs doublet models.

### B. Self-couplings of the Higgs bosons (sleptons)

It is a straightforward calculation by inserting Eqs. (17, 19, 20, 26, 27) into Eqs. (6), to obtain the desired interaction terms. Similar to the interactions of gauge-Higgs (slepton) bosons, we split the Lagrangian into pieces:

$$\mathcal{L}^S_{int} = \mathcal{L}_{SSS} + \mathcal{L}_{SSSS} \tag{44}$$

where $\mathcal{L}_{SSS}$ represents trilinear coupling terms, and $\mathcal{L}_{SSSS}$ four scalar boson coupling terms. The trilinear pieces are most interesting. If the masses of the scalars are appropriate, the decays of one Higgs boson into other two Higgs bosons may occur. After tedious computation, we obtain:

$$\begin{aligned}\mathcal{L}_{SSS} = & -\frac{g^2+g'^2}{8}A^{ij}_{even}B^k_{even}H_iH^0_jH^0_k - \frac{g^2+g'^2}{8}A^{ij}_{odd}B^k_{even}H_{5+i}H^0_{5+j}H^0_k \\ & -A^{kij}_{ec}H^0_kH^-_iH^+_j + iA^{kij}_{oc}H^0_{5+k}H^-_iH^+_j\end{aligned} \tag{45}$$

and

$$\begin{aligned}\mathcal{L}_{SSSS} = & -\frac{g^2+g'^2}{32}A^{ij}_{even}A^{kl}_{even}H_iH^0_jH^0_kH^0_l - \frac{g^2+g'^2}{32}A^{ij}_{odd}A^{kl}_{odd}H_{5+i}H^0_{5+j}H^0_{5+k}H^0_{5+l} \\ & -\frac{g^2+g'^2}{16}A^{ij}_{even}A^{kl}_{odd}H_iH^0_jH^0_{5+k}H^0_{5+l} - \mathcal{A}^{klij}_{ec}H^0_kH^0_lH^-_iH^+_j \\ & -\mathcal{A}^{klij}_{oc}H^0_{5+k}H^0_{5+l}H^-_iH^+_j - i\mathcal{A}^{klij}_{eoc}H^0_kH^0_{5+l}H^-_iH^+_j - \mathcal{A}^{klij}_{cc}H^-_kH^+_lH^-_iH^+_j\end{aligned} \tag{46}$$

with

$$A^{ij}_{even} = \sum_{\alpha=1}^{5} Z^{i\alpha}_{even} Z^{j\alpha}_{even} - 2 Z^{i2}_{even} Z^{j2}_{even},$$
$$A^{ij}_{odd} = \sum_{\alpha=1}^{5} Z^{i\alpha}_{odd} Z^{j\alpha}_{odd} - 2 Z^{i2}_{odd} Z^{j2}_{odd},$$
$$B^{i}_{even} = \upsilon_1 Z^{i1}_{even} - \upsilon_2 Z^{i2}_{even} + \sum_I \upsilon_{\tilde{\nu}_I} Z^{iI+2}_{even}. \tag{47}$$

The definitions of $A^{kij}_{ec}$, $A^{kij}_{oc}$, $\mathcal{A}^{klij}_{ec}$, $\mathcal{A}^{klij}_{oc}$, $\mathcal{A}^{klij}_{eoc}$ and $\mathcal{A}^{ijkl}_{cc}$ are lengthy, so we put them in Appendix C. The Feynman rules are summarized in Fig.5 and Fig. 6. Note that the lepton number violations have led to very complicated form for the $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$.

### C. The R-parity violation couplings of Higgs

In this subsection we compute the R-parity violation couplings of Higgs i.e. the Higgs couplings to charginos (charged lepton) and neutralinos (neutrinos). After spontaneous breaking of the gauge symmetry SU(2)×U(1), the gauginos, higgsinos and leptons with the same electric charge may mix as described in Section II. Let us proceed to compute the interaction $S\tilde{\kappa}^0_i\tilde{\kappa}^0_j$ (Higgs-neutralinos-neutralinos interactions).

The interactions (in two-component spinors) [2] are:

$$\begin{aligned}\mathcal{L}_{S\kappa\kappa} = & i\sqrt{2}g\Big(H^{1\dagger}\frac{\tau^i}{2}\lambda^i_A\psi_{H^1} - \bar{\psi}_{H^1}\frac{\tau^i}{2}\bar{\lambda}^i_A H^1\Big) - i\sqrt{2}g'\Big(\frac{1}{2}H^{1\dagger}\psi_{H^1}\lambda_B - \frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^1}H^1\Big) \\ & + i\sqrt{2}g\Big(H^{2\dagger}\frac{\tau^i}{2}\lambda^i_A\psi_{H^2} - \bar{\psi}_{H^2}\frac{\tau^i}{2}\bar{\lambda}^i_A H^2\Big) + i\sqrt{2}g'\Big(\frac{1}{2}H^{2\dagger}\psi_{H^2}\lambda_B - \frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^2}H^2\Big) \\ & + i\sqrt{2}\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}\lambda^i_A\psi_{L^I} - \frac{1}{2}g'\lambda_B\psi_{L^I}\Big) - i\sqrt{2}\tilde{L}^I\Big(g\frac{\tau^i}{2}\bar{\lambda}^i_A\bar{\psi}_{L^I} - \frac{1}{2}g'\bar{\lambda}_B\bar{\psi}_{L^I}\Big) \\ & + i\sqrt{2}g'\tilde{R}^{I\dagger}\lambda_B\psi_{R^I} - i\sqrt{2}g'\tilde{R}^I\bar{\lambda}_B\bar{\psi}_{R^I} - \frac{1}{2}l_I\varepsilon_{ij}\Big(\psi^i_{H^1}\psi^j_{L^I}\tilde{R}^I + \psi^i_{H^1}\psi_{R^I}\tilde{L}^I_j \\ & + \psi_{R^I}\psi^j_{L^I}H^1_i + h.c.\Big) - \frac{1}{2}\lambda_{IJK}\varepsilon_{ij}\Big(\psi^i_{L^I}\psi^j_{L^J}\tilde{R}^K + \psi^i_{L^I}\psi_{R^K}\tilde{L}^J_j \\ & + \psi^j_{L^J}\psi_{R^K}\tilde{L}^I_i + h.c.\Big)\ .\end{aligned} \tag{48}$$

We sketch the derivation for the vertices, $S\tilde{\kappa}^0_i\tilde{\kappa}^0_j$ etc. Starting with the Eq. (48), we convert the pieces from the two-component spinor notation into four-component spinor notation first, then using the definitions in Eq. (33 $\sim$ 36) and Eq. (41), we obtain:

$$\mathcal{L}_{S\kappa\kappa} = \frac{\sqrt{g^2+g'^2}}{2}\Big[C^{ijm}_{snn}H_i\bar{\kappa}^0_j P_L\kappa^0_m + C^{ijm*}_{snn}H_i\bar{\kappa}^0_j P_R\kappa^0_m\Big]$$
$$+\frac{g}{\sqrt{2}}\Big[C^{ijm}_{skk}H_i\bar{\kappa}^+_m P_L\kappa^+_j + C^{ijm*}_{skk}H_i\bar{\kappa}^+_j P_R\kappa^+_m\Big]$$
$$+i\frac{\sqrt{g^2+g'^2}}{2}\Big[C^{ijm}_{onn}H_{5+i}\bar{\kappa}^0_j P_R\kappa^0_m - C^{ijm*}_{onn}H_{5+i}\bar{\kappa}^0_m P_L\kappa^0_j\Big]$$
$$+i\frac{g}{\sqrt{2}}\Big[C^{ijm}_{okk}H_{5+i}\bar{\kappa}^+_m P_L\kappa^+_j - C^{ijm*}_{okk}H_{5+i}\bar{\kappa}^+_m P_R\kappa^+_j\Big]$$
$$+\sqrt{g^2+g'^2}\Big[C^{ijm}_{Lnk}\bar{\kappa}^+_j P_L\kappa^0_m H^+_i - C^{ijm}_{Rnk}\bar{\kappa}^+_j P_R\kappa^0_m H^+_i\Big] \tag{49}$$

and the coefficients $C^{ijm}_{snn}$, $C^{ijm}_{Lnk}$, $C^{ijm}_{Rnk}$ and $C^{ijm}_{skk}$, being lengthy, are put in Appendix D Here $P_{L,R} = \frac{1\pm\gamma_5}{2}$ are project operators and the transformation matrices $Z_\pm$, $Z_N$ are defined in Sect.II. The corresponding Feynman rules are summarized in Fig. 7. Note here: as for $\kappa^0_i$ being a Majorana fermion, the useful identity

$$\bar{\kappa}^0_j(1\pm\gamma_5)\kappa^0_k = \bar{\kappa}^0_k(1\pm\gamma_5)\kappa^0_j, \tag{50}$$

holds for the anticommuting four-component Majorana spinors always, that the $H_i\bar{\kappa}^0_j\kappa^0_k$ interaction can be rearranged symmetrically under the interchange of the indices $j$ and $k$.

Since $\nu_e$ $(e)$, $\nu_\mu$ $(\mu)$ and $\nu_\tau$ $(\tau)$ should be identified with the lightest three 'neutralinos' ('charginos') in the model, there must be some fresh and interesting phenomena relevant to them, e.g. $\kappa^0_i$ $(i=1,2,3,4) \to \tau H^+_j$ ($j$ = 2,3, $\cdots$, 8), $\kappa^0_i$ $(i=1,2,3,4) \to \nu_{e,\mu,\tau}H^0_j$ ($j$ = 1, 2, $\cdots$, 5) etc may occur, if the masses are suitable (the phase space is allowed). Namely, these interactions without R-parity conservation may induce new rare processes [8,9,11,12,15,19].

### D. The R-parity violation couplings of gauge bosons

In this subsection we focus on the R-parity violation couplings of the gauge bosons ($W$, $Z$, $\gamma$) i.e. the couplings of the gauge bosons ($W$, $Z$, $\gamma$) to the charginos (charged leptons) and neutralinos (neutrinos). Since we identify the three types of charged leptons (neutrinos) with the three lightest charginos (neutralinos), the restrictions relating to them from the present experiments must be considered carefully. The relevant interactions come from the following pieces of Lagrangian:

$$
\mathcal{L}_{int}^{gcn} = -i\bar{\lambda}_A^i \bar{\sigma}^\mu \mathcal{D}_\mu \lambda_A^i - i\bar{\lambda}_B \bar{\sigma}^\mu \mathcal{D}_\mu \lambda_B - i\bar{\psi}_{H^1} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{H^1} - i\bar{\psi}_{H^2} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{H^2} - i\bar{\psi}_{L^I} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{L^I}
$$

$$
-i\bar{\psi}_{R^I} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{R^I} \tag{51}
$$

with

$$
\begin{aligned}
\mathcal{D}_\mu \lambda_A^1 &= \partial_\mu \lambda_A^1 - g A_\mu^2 \lambda_A^3 + g A_\mu^3 \lambda_A^2, \\
\mathcal{D}_\mu \lambda_A^2 &= \partial_\mu \lambda_A^2 - g A_\mu^3 \lambda_A^1 + g A_\mu^1 \lambda_A^3, \\
\mathcal{D}_\mu \lambda_A^3 &= \partial_\mu \lambda_A^3 - g A_\mu^1 \lambda_A^2 + g A_\mu^2 \lambda_A^1, \\
\mathcal{D}_\mu \lambda_B &= \partial_\mu \lambda_B, \\
\mathcal{D}_\mu \psi_{H^1} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} - \frac{i}{2} g' B_\mu) \psi_{H^1}, \\
\mathcal{D}_\mu \psi_{H^2} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} + \frac{i}{2} g' B_\mu) \psi_{H^2}, \\
\mathcal{D}_\mu \psi_{L^I} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} - \frac{i}{2} g' B_\mu) \psi_{L^I}, \\
\mathcal{D}_\mu \psi_{R^I} &= (\partial_\mu + ig' B_\mu) \psi_{R^I} \ .
\end{aligned} \tag{52}
$$

Similar to the couplings in $\mathcal{L}_{S\kappa\kappa}$, we convert all spinors in Eq. (51) into four component ones and with Eq. (36) and Eq. (41), then we obtain:

$$
\begin{aligned}
\mathcal{L}_{int}^{gcn} = &\Big\{ \sqrt{g^2 + g'^2} \sin\theta_W \cos\theta_W A_\mu \bar{\kappa}_i^+ \gamma^\mu \kappa_i^+ - \sqrt{g^2 + g'^2} Z_\mu \bar{\kappa}_i^+ \Big[ \cos^2\theta_W \delta_{ij} \gamma^\mu \\
&+ \frac{1}{2} \Big( Z_-^{*i2} Z_-^{j2} + \sum_{I=1}^{3} Z_-^{*i2+I} Z_-^{j2+I} \Big) \gamma^\mu P_R \\
&+ \Big( \frac{1}{2} Z_+^{*i2} Z_+^{j2} - \sum_{I=1}^{3} Z_+^{*i2+I} Z_+^{j2+I} \Big) \gamma^\mu P_L \Big] \kappa_j^+ \Big\} \\
&+ \Big\{ g \bar{\kappa}_j^+ \Big[ \Big( - Z_+^{*i1} Z_N^{j2} + \frac{1}{\sqrt{2}} Z_+^{*i2} Z_N^{j4} \Big) \gamma^\mu P_L + \Big( Z_N^{*i2} Z_-^{j1} \\
&+ \frac{1}{\sqrt{2}} \Big( Z_N^{*i3} Z_-^{j2} + \sum_{I=1}^{3} Z_N^{*i4+I} Z_-^{j2+I} \Big) \Big) \gamma^\mu P_R \Big] \kappa_i^0 W_\mu^+ + h.c. \Big\} \\
&+ \frac{\sqrt{g^2 + g'^2}}{2} \bar{\kappa}_i^0 \gamma^\mu \Big\{ \Big[ Z_N^{*i4} Z_N^{j4} - \Big( Z_N^{*i3} Z_N^{j3} + \sum_{\alpha=5}^{7} Z_N^{*i\alpha} Z_N^{j\alpha} \Big) \Big] P_L \\
&- \Big[ Z_N^{*i4} Z_N^{j4} - \Big( Z_N^{*i3} Z_N^{j3} + \sum_{\alpha=5}^{7} Z_N^{*i\alpha} Z_N^{j\alpha} \Big) \Big] P_R \Big\} \kappa_j^0 Z_\mu \ .
\end{aligned} \tag{53}
$$

The corresponding Feynman rules are summarized in Fig. 8. Since we identify the three lightest neutralinos (charginos) with three types of neutrinos (charged leptons), some fea-

tures about Eq. (53) should be emphasized:

- At tree level for the $\gamma$-$\kappa$-$\kappa$ vertices, there is no lepton flavor-changing current interaction, that is the same as that in the SM and MSSM with R-parity.
- At tree level for the $Z$-$\kappa$-$\kappa$ vertices, there are lepton flavor-changing current interactions, that is different from the MSSM with R-parity.
- Similar to the $Z$-$\kappa$-$\kappa$ vertices, there are the vertices such as $W\tau\nu_e$, which are forbidden in the MSSM with R-parity.

### E. The R-parity violation couplings of quarks and/or squarks

In this subsection we focus the R-parity violation couplings of quarks and/or squarks i.e. pursue the Feynman rules for the interactions of quarks and scalar-quarks with charginos (charged leptons) and neutralinos (neutrinos) e.g. the $\tilde{Q}q\kappa_i^{\pm}$ vertices. Because the mixing of neutrinos (charged leptons) and original neutralinos (charginos), so the vertices will lead to certain interesting phenomenology, thus it is interesting to write them down precisely. Of the vertices, they can be divided into two categories: the supersymmetric analogies of the $q\bar{q}W^{\pm}$ and $q\bar{q}Z$ interactions and the supersymmetric analogy of the $q\bar{q}H$ interaction which is proportional to quark mass and depends on the properties of the Higgs bosons in the model.

Let us consider the $\bar{q}q\kappa_i^{\pm}$ interactions first. In two-component spinors they are as the follows:

$$\begin{aligned}\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} = ig\Big(C^{IJ}\tilde{Q}_2^{I*}\lambda_A^{-}\psi_{Q_1}^J + C^{IJ*}\tilde{Q}_1^{J*}\lambda_A^{+}\psi_{Q_2}^I\Big) - ig\Big(C^{IJ*}\tilde{Q}_2^{I}\bar{\lambda}_A^{-}\bar{\psi}_{Q_1}^J + C^{IJ}\tilde{Q}_1^{J}\bar{\lambda}_A^{+}\bar{\psi}_{Q_2}^I\Big) \\ -\frac{d^I}{2}\Big(C^{IJ}\psi_{H^1}^2\psi_{Q^1}^J\tilde{D}^I + C^{IJ}\psi_{H^1}^2\tilde{Q}_1^J\psi_D^I + h.c.\Big) + \frac{u^I}{2}\Big(C^{JI*}\psi_{H^2}^1\psi_{Q^2}^J\tilde{U}^I \\ +C^{JI*}\psi_{H^2}^1\psi_U^I\tilde{Q}_2^J + h.c.\Big) - \frac{1}{2}\lambda_{IJK}\Big(C^{JK}\psi_{L^I}^2\psi_{Q^1}^K\tilde{D}^J \\ +C^{JK}\psi_{L^I}^2\tilde{Q}_1^K\psi_D^J + h.c.\Big)\,. \end{aligned} \tag{54}$$

As discussed above, we convert the two-component spinors into four-component spinors:

$$
\begin{aligned}
\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} = C^{IJ}\bar{\kappa}_j^{+}\Big\{\Big(&-gZ_{D_I}^{i1}Z_{-}^{j1}+\frac{d^I}{2}Z_{D_I}^{i2}Z_{-}^{j2}+\frac{1}{2}\lambda'_{KIJ}Z_{D_I}^{i2}Z_{-}^{j2+K}\Big)P_L \\
&+\frac{u^J}{2}Z_{+}^{j2*}Z_{D_I}^{i1}P_R\Big\}\psi_{u^I}\tilde{D}_{Ii}^{+}+C^{IJ*}\bar{\kappa}_j^{-}\Big\{\Big(-gZ_{U_J}^{i1}Z_{+}^{j1}+\frac{u^J}{2}Z_{U_J}^{i2}Z_{+}^{j2}\Big)P_L \\
&-\Big(\frac{d^I}{2}Z_{U_J}^{j1*}Z_{-}^{j2*}+\frac{1}{2}\lambda'_{KIJ}Z_{U_J}^{j1*}Z_{-}^{j2+K*}\Big)P_R\Big\}\psi_{d^J}\tilde{U}_{Ji}^{-}+h.c.\ . \qquad (55)
\end{aligned}
$$

Here $\psi_{u^I}$, $\psi_{d^I}$ are four-component quark spinors of the I-th generation. The $\kappa_j^-$ is a charged-conjugate state of $\kappa_j^+$, and $\kappa_j^+$ is defined by Eq. (41).

For the $\tilde{Q}q\kappa_i^0$ interactions, in two-component notation they are:

$$
\begin{aligned}
\mathcal{L}_{\tilde{Q}q\kappa_i^0} = &\, i\sqrt{2}\tilde{Q}^{I*}\left(g\frac{\tau^3}{2}\lambda_A^3+\frac{1}{6}g'\lambda_B\right)\psi_Q^I - i\sqrt{2}\tilde{Q}^I\left(g\frac{\tau^3}{2}\bar{\lambda}_A^3+\frac{1}{6}g'\bar{\lambda}_B\right)\bar{\psi}_Q^I \\
&-i\frac{2\sqrt{2}}{3}g'\tilde{U}^{I*}\lambda_B\psi_U^I+i\frac{2\sqrt{2}}{3}g'\tilde{U}^I\bar{\lambda}_B\bar{\psi}_U^I+i\frac{\sqrt{2}}{3}g'\tilde{D}^{I*}\lambda_B\psi_D^I-i\frac{\sqrt{2}}{3}g'\tilde{D}^I\bar{\lambda}_B\bar{\psi}_D^I \\
&+\frac{d^I}{2}\left\{\psi_{H^1}^1\psi_{Q^2}^I\tilde{D}^I+\psi_{H^1}^1\psi_D^I\tilde{Q}_2^I+h.c.\right\}-\frac{u^I}{2}\left\{\psi_{H^2}^2\psi_{Q^1}^I\tilde{U}^I+\psi_{H^2}^2\psi_U^I\tilde{Q}_1^I+h.c.\right\} \\
&+\frac{1}{2}\lambda'_{IJJ}\left\{\psi_{L^I}^1\psi_{Q^2}^J\tilde{D}^J+\psi_{L^I}^1\psi_D^J\tilde{Q}_2^J+h.c.\right\}\ . \qquad (56)
\end{aligned}
$$

After converting into four-component notation straightforward and using the definition for neutralino mass eigenstates, we have:

$$
\begin{aligned}
\mathcal{L}_{\tilde{Q}q\kappa_i^0} = \kappa_j^0\Bigg\{\Bigg[&\frac{e}{\sqrt{2}\sin\theta_W\cos\theta_W}Z_{U^I}^{i1*}\left(\cos\theta_W Z_N^{i2}+\frac{1}{3}\sin\theta_W Z_N^{j1}\right)-\frac{u^I}{2}Z_{U^I}^{i1*}Z_N^{j4*}\Bigg]P_L \\
&+\left[\frac{2\sqrt{2}}{3}g'Z_{U^I}^{i2*}Z_N^{j1}-\frac{u^I}{2}Z_{U^I}^{i1*}Z_N^{j4*}\right]P_R\Bigg\}\psi_{u^I}\tilde{U}_{I,i}^{-}+\bar{\kappa}_j^0\Bigg\{\Bigg[\frac{e}{\sqrt{2}\sin\theta_W\cos\theta_W}Z_{D^I}^{i1} \\
&\left(-\cos\theta_W Z_N^{i2}+\frac{1}{3}\sin\theta_W Z_N^{j1}\right)+\frac{d^I}{2}Z_{D^I}^{i2}Z_N^{j3}+\frac{1}{2}\lambda_{KIJ}Z_{D^I}^{i2}Z_N^{j4+K}\Bigg]P_L \\
&+\left[-\frac{\sqrt{2}}{3}g'Z_{D^I}^{i2}Z_N^{j1}+\frac{d^I}{2}Z_{D^I}^{i1*}Z_N^{j3*}+\frac{1}{2}\lambda_{KIJ}Z_{D^I}^{i1*}Z_N^{j4+K*}\right]P_R\Bigg\}\psi_{d^I}\tilde{D}_{Ii}^{+}+h.c.\ . \qquad (57)
\end{aligned}
$$

Thus the Feynman rules for the concerned interactions may be depicted exactly as the last two diagrams in Fig. 9.

## IV. VARIOUS R-PARITY BREAKING MODELS AND THE FREEDOM FOR REDEFINING THE SUPERFIELDS

As stated at beginning, we are working in a very general model, where the R-parity is broken via various lepton-number violations with possible parameters explicitly and spon-

taneously, then an interesting problem is raised. Namely, we should realize possible freedom in representing the model and should fix it properly. In fact, there is some confusion on the freedom in literature. In this section we focus the problem carefully.

If the superpotential and the soft SUSY breaking were switched off, the MSSM models would turn to have a $U(n+1)$ global symmetry, i.e. in the case the 'down' type of Higgs boson $H^1$ and the leptons chiral superfields $L^I (I = e, \mu, \tau; n = 3)$ can be composed as a 'vector', and under a transformation as $(H^1, L^I) \to U \cdot (H^1, L^I)$, $U \in U(n{+}1 = 4)$, the theory would be invariant. Due to the invariance, the quantum numbers of leptons would become meaningless. Whereas the $U(4)$ symmetry is completely broken down when switching on the superpotential and the SUSY breaking terms as well, then two possibilities happen: a) If the superpotential and the soft SUSY breaking terms in the Lagrangian of the model conserve each lepton number respectively as a global symmetry, the lepton quantum numbers are fixed so the lepton numbers make senses. In fact this case is just the MSSM with R-parity. b) If they are switched on, but break the lepton numbers, although the $U(4)$ symmetry is lost, instead a freedom to re-define the three lepton superfields and the down type Higgs is raised in representing the model. Namely if all the terms in the superpotential and in the soft SUSY breaking terms undergo a $U(4)$ transformation accordingly, i.e. the $U(4)$ acts as redefining the lepton and Higgs superfields in the model, then superficially the VEVs of the sneutrinos, the mass matrices and the relevant couplings are changed accordingly, whereas the physics is not changed. The MSSMs without R-parity may be constructed at beginning with very different parameters even very different assumptions naively, but they may be equivalent exactly i.e. they are just the same one of the models. Indeed in the case b) with such broken lepton-number superfields and complicated soft SUSY-breaking terms, it is not so straightforward to see the freedom, so it is an important and non-trivial task. Let us examine the problem now.

To bare the task to realize the freedom for the MSSM without R-parity in mind, in the section we precisely show the equivalence of the models which are related to each other by $U(n+1)$ ($n$ is the family number with broken lepton number) transformations. Namely

the $U(n+1)$ transformations can be understood as a freedom for re-defining the relevant superfields. Finally in this section we propose two suggestions which may be considered as 'conventions' for possible choices to fix the freedom. Of the two, one is 'to rotate away' the nonzero vacuum expectation values (VEVs) of all the three generations of sneutrinos, this is emphasized by Refs. [15,20,21], the other one is to 'rotate away' the bilinear terms in the superpotential [5,8,9,12,16]. Note that in the second case, in general, the bilinear R-parity violation terms in soft breaking SUSY terms so nonzero VEVs of the sneutrinos still may exist.

We would emphasize here that based on the whole effective Lagrangian we may compute the spectrum (mixing) of the content particles of the model precisely, and may have a global view of the model too. Furthermore, having the precise Lagrangian one may easy connect the effective one to a more fundamental theory, thus we would not do the problem such as in the references [4] where from the very beginning only the 'basis-indepedent' parameters are focused on so as to investigate the phenomenology of the model. Whereas in the present way, we should examine the freedom in defining the fields carefully, and make all the parameters being substantial. In fact, when all possible terms (not only in superpotential but also in SUSY soft-breaking and D-terms) are involved the problem is not so transparent to realize the freedom.

Here we take the general case, that the three lepton-numbers are broken, as an example to examine the freedom. In fact, the freedom is $U(n+1=4)$ ($n$: the number of violated lepton-numbers) in defining the superfields within the extented MSSM as shown in the follows.

The $U(4)$

$$X^{\dagger}X = XX^{\dagger} = I, \tag{58}$$

and $X \in U(4)$ is $4 \times 4$ matrix, $I$ is the unit matrix. The $U(4)$ matrix 'acting' on the 'old' Higgs and lepton superfields $\hat{H}^1, \hat{L}^1, \hat{L}^2, \hat{L}^3$ is defined as:

$$\begin{pmatrix} \hat{H}^{a^1} \\ \hat{L}^{a^1} \\ \hat{L}^{a^2} \\ \hat{L}^{a^3} \end{pmatrix} = X \begin{pmatrix} \hat{H}^1 \\ \hat{L}^1 \\ \hat{L}^2 \\ \hat{L}^3 \end{pmatrix}, \tag{59}$$

where $\hat{H}^{a^1}, \hat{L}^{a^1}, \hat{L}^{a^2}, \hat{L}^{a^3}$ are 'new' Higgs-lepton superfields. As the D-terms in the model are invariant under the $U(4)$ transformation, so we need not consider them at all for present purpose. As for the superpotential, having the $U(4)$ transformation performed, it turns into the following form accordingly by means of the 'new' superfields (with a upper-suffix as in Eq. (59)):

$$\begin{aligned} W = & \varepsilon_{ij}\mu^a \hat{H}_i^{a^1}\hat{H}_j^2 + \varepsilon_{ij}\epsilon_I^a \hat{H}_i^2 \hat{L}_j^{a^I} + \varepsilon_{ij} h_{IJ}^l \hat{H}_i^{a^1} \hat{L}_j^{a^I} \hat{R}^J \\ & - u_I(\hat{H}_1^2 C^{JI*}\hat{Q}_2^J - \hat{H}_2^2 \hat{Q}_1^J \delta^{IJ})\hat{U}^I \\ & - h_{IJ}^d(\hat{H}_1^{a^1}\hat{Q}_2^J\delta^{IJ} - \hat{H}_2^{a^1}\hat{Q}_1^J C^{IJ})\hat{D}^I + \varepsilon_{ij}\lambda_{IJK}^a \hat{L}_i^{a^I}\hat{L}_j^{a^J}\hat{R}^K \\ & + \lambda_{IJK}^{a'}(\hat{L}_1^{a^K}\hat{Q}_2^J \delta IJ - \hat{L}_2^{a^K}\hat{Q}_1^J C^{IJ})\hat{D}^I + \lambda''_{IJK}\hat{U}^I\hat{D}^J\hat{D}^K, \end{aligned} \tag{60}$$

and the 'new' coefficients are related to the 'old' ones through the $U(4)$ matrix elements precisely:

$$\begin{aligned} h_{IJ}^l = \; & l_J(X_{11}^* X_{(I+1)(J+1)}^* - X_{(I+1)1}^* X_{1(J+1)}^*) \\ & + \sum_{KM} \lambda_{KMJ}(X_{1(K+1)}^* X_{(I+1)(M+1)}^* \\ & - X_{(I+1)(K+1)}^* X_{1(M+1)}^*), \end{aligned} \tag{61}$$

$$h_{IJ}^d = \; d_J X_{11}^* - \sum_K \lambda'_{KIJ} X_{1(K+1)}^*, \tag{62}$$

$$\begin{aligned} \lambda_{IJK}^a = \; & l_K X_{(I+1)1}^* X_{(J+1)(K+1)}^* \\ & + \sum_{MN} \lambda_{MNK} X_{(I+1)(M+1)}^* X_{(J+1)(N+1)}^*, \end{aligned} \tag{63}$$

$$\lambda_{IJK}^{a'} = -d_I X_{(I+1)1}^* + \sum_M \lambda'_{MKJ} X_{(I+1)(M+1)}^*, \tag{64}$$

$$h_I^a = \; l_I X_{11}^* X_{1(I+1)}^* + \sum_{JK} \lambda_{KJI} X_{1(K+1)}^* X_{1(J+1)}^*, \tag{65}$$

$$\mu^a = \; X_{11}^* \mu - \sum_I \epsilon_I X_{1(I+1)}^*, \tag{66}$$

$$\epsilon_I^a = \quad -X^*_{(I+1)1}\mu + \sum_J X^*_{(I+1)(J+1)}\epsilon_J. \tag{67}$$

Here $X^*_{\alpha\beta}(\alpha,\beta = 1,2,3,4)$ are the complex conjugations of $X_{\alpha\beta}$, and $(I,J,K,M,N = 1,2,3)$ always. As the same way, the soft SUSY-breaking terms in Lagrangian now correspondingly become:

$$\begin{aligned}
\mathcal{L}_{soft} = &-M^{s^2}_{11}H_i^{a^1*}H_i^{a^1} - \sum_I M^{s^2}_{1(I+1)}(H_i^{a^1*}\tilde{L}_i^{a^I} + H_i^{a^1}\tilde{L}_i^{a^I*}) \\
&-\sum_{IJ} M^{s^2}_{(I+1)(J+1)}\tilde{L}_i^{a^I*}\tilde{L}_i^{a^J} - m^2_{R^I}\tilde{R}^{I*}\tilde{R}^I - m^2_{Q^I}\tilde{Q}_i^{I*}\tilde{Q}_i^I \\
&-m^2_{D^I}\tilde{D}^{I*}\tilde{D}^I - m^2_{U^I}\tilde{U}^{I*}\tilde{U}^I + (m_1\lambda_B\lambda_B + m_2\lambda_A^i\lambda_A^i \\
&+m_3\lambda_G^a\lambda_G^a + h.c.) + \{\varepsilon_{ij}B^aH_i^{a^1}H_j^2 + \varepsilon_{ij}B_I^aH_i^2\tilde{L}_j^{a^I} \\
&+\varepsilon_{ij}l^a_{s_{IJ}}H_i^{a^1}\tilde{L}_j^{a^I}\tilde{R}^J - d^a_{s_{IJ}}(H_1^{a^1}\tilde{Q}_2^J\delta^{IJ} - H_2^{a^1}\tilde{Q}_1C^{IJ})\tilde{D}^I \\
&+u_{s_I}(-C^{KI*}H_1^2\tilde{Q}_2^I + H_2^2\tilde{Q}_1^I)\tilde{U}^I + \varepsilon_{ij}\lambda^{s_a}_{IJK}\tilde{L}_i^{a^I}\tilde{L}_i^{a^J}\tilde{R}^K \\
&+\lambda^{s'_a}_{KIJ}(\tilde{L}_1\tilde{Q}_2^J\delta^{IJ} - \tilde{L}_2^{a^K}\tilde{Q}_1^JC^{IJ})\tilde{D}^I \\
&+\lambda^{s''}_{IJK}\tilde{U}^I\tilde{D}^J\tilde{D}^K + h.c.\}.
\end{aligned} \tag{68}$$

Here the 'new' soft breaking parameters are defined as

$$\begin{aligned}
M^2_{s_{\alpha\beta}} = &\, m^2_{H^1}X^*_{\alpha 1}X^*_{\beta 1} + \sum_I m^2_{L^I}X^*_{\alpha(I+1)}X^*_{\beta(I+1)} + \sum_{I\neq J} m^2_{L^{IJ}}X^*_{\alpha(I+1)}X^*_{\beta(J+1)} \\
&+\sum_I m^2_{HL^I}X^*_{\alpha 1}X^*_{\beta(I+1)},
\end{aligned} \tag{69}$$

$$B^a = \quad BX^*_{11} - \sum_I B_I X^*_{1(I+1)}, \tag{70}$$

$$B_I^a = \quad -BX^*_{(I+1)1} + \sum_J B_J X^*_{(I+1)(J+1)}, \tag{71}$$

$$\begin{aligned}
l^a_{s_{MK}} = &\sum_{s_I}(-X^*_{(M+1)1}X^*_{1(I+1)} + X^*_{11}X^*_{(M+1)(I+1)})\delta_{IK} \\
&+\sum_{IJ}\lambda^s_{IJK}(X^*_{(M+1)(I+1)}X^*_{1(J+1)} + X^*_{1(I+1)}X^*_{(M+1)(J+1)},
\end{aligned} \tag{72}$$

$$d^a_{s_{IJ}} = \, d_{s_I}X^*_{11} - \sum_K \lambda^{s'}_{KIJ}X^*_{1(K+1)}, \tag{73}$$

$$\begin{aligned}
\lambda^{s_a}_{IJK} = &\, l_{s_K}X^*_{(I+1)1}X^*_{(J+1)(K+1)} \\
&+\sum_{MN}\lambda^s_{MNK}X^*_{(I+1)(M+1)}X^*_{(J+1)(N+1)},
\end{aligned} \tag{74}$$

$$\lambda^{s'_a}_{KIJ} = -d_{s_I}X^*_{(K+1)1} + \sum_M \lambda^{s'}_{MJI}X^*_{(K+1)(M+1)}, \tag{75}$$

$$h_I^{s_a} = \ l_{s_I} X_{11}^* X_{1(I+1)}^* + \sum_{JK} \lambda_{KJI} X_{1(K+1)}^* X_{1(J+1)}^*. \tag{76}$$

Now, the $H^{a^1}$ and slepton acquire vacuum expectation values (VEVs):

$$H^{a^1} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_1^a + \upsilon_1^a + i\phi_1^a) \\ H_2^{a^1} \end{pmatrix} \tag{77}$$

$$H^2 = \begin{pmatrix} H_1^2 \\ \frac{1}{\sqrt{2}}(\chi_2^0 + \upsilon_2 + i\phi_2^0) \end{pmatrix} \tag{78}$$

and

$$\tilde{L}^{a^I} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_{\tilde{\nu}_I}^a + \upsilon_{\tilde{\nu}_I}^a + i\phi_{\tilde{\nu}_I}^a) \\ \tilde{L}_2^{a^I} \end{pmatrix} \tag{79}$$

where

$$\begin{aligned} \upsilon_1^a &= X_{11}\upsilon_1 + \sum_J X_{1(J+1)}\upsilon_{\tilde{\nu}_J}, \\ \upsilon_{\tilde{\nu}_I}^a &= X_{(I+1)1}\upsilon_1 + \sum_J X_{(I+1)(J+1)}\upsilon_{\tilde{\nu}_J}. \end{aligned} \tag{80}$$

As before by expanding the scalar potential, the tadpole terms become:

$$V_{tadpole} = t_1^0\chi_1^0 + t_2^0\chi_2^0 + t_{\tilde{\nu}_1}^0\chi_{\tilde{\nu}_1}^0 + t_{\tilde{\nu}_2}^0\chi_{\tilde{\nu}_2}^0 + t_{\tilde{\nu}_3}^0\chi_{\tilde{\nu}_3}^0 \tag{81}$$

where

$$\begin{aligned} t_1^a &= \frac{1}{8}(g^2 + g^{'2})\upsilon_1^a(\upsilon_1^{a^2} - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^{a^2}) + \mu^2\upsilon_1^a - B^a\upsilon_2 \\ &\quad - \sum_I \mu^a\epsilon_I^a\upsilon_{\tilde{\nu}_I}^a + M_{s_{11}}^2\upsilon_1^a + \sum_I M_{s_{1(I+1)}}^2\upsilon_{\tilde{\nu}_I}^a \\ t_2^a &= -\frac{1}{8}(g^2 + g^{'2})\upsilon_2(\upsilon_1^{a^2} - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^{a^2}) + \mu^{a^2}\upsilon_2 - B^a\upsilon_1^a \\ &\quad + \sum_I \epsilon_I^{a^2}\upsilon_2 + \sum_I B_I^a\upsilon_{\tilde{\nu}_I}^a + m_{H^2}^2\upsilon_2, \\ t_{\tilde{\nu}_I}^a &= \frac{1}{8}(g^2 + g'^2)\upsilon_{\tilde{\nu}_I}^a(\upsilon_1^{a^2} - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^{a^2}) + \epsilon_I^a\sum_J \epsilon_J^a\upsilon_{\tilde{\nu}_J}^a \\ &\quad -\mu^a\epsilon_I^a\upsilon_1^a + B_I^a\upsilon_2 + M_{s_{(I+1)(I+1)}}^2\upsilon_{\tilde{\nu}_I}^a + M_{s_{1(I+1)}}^2\upsilon_1^a + \\ &\quad \sum_{J\neq I} M_{s_{(I+1)(J+1)}}^2\upsilon_{\tilde{\nu}_J}^a. \end{aligned} \tag{82}$$

Namely in the new basis $(\phi_1^a, \phi_2^0, \phi_{\tilde{\nu}_1}^a, \phi_{\tilde{\nu}_2}^a, \phi_{\tilde{\nu}_3}^a)$, the mass matrix of the CP-odd Higgs becomes

$$M_{odd}^{a^2} = \begin{pmatrix} s_{11}^a & B^a & M_{s_{12}}^2 - \mu^a\epsilon_1^a & M_{s_{13}}^2 - \mu^a\epsilon_2^a & M_{s_{14}}^2 - \mu^a\epsilon_3^a \\ B^a & s_{22}^a & -B_1^a & -B_2^a & -B_3^a \\ M_{s_{12}}^2 - \mu^a\epsilon_1^a & -B_1^a & s_{33}^a & \epsilon_1^a\epsilon_2^a + M_{s_{23}}^2 & \epsilon_1^a\epsilon_3^a + M_{s_{24}}^2 \\ M_{s_{13}}^2 - \mu^a\epsilon_2^a & -B_2^a & \epsilon_1^a\epsilon_2^a + M_{s_{23}}^2 & s_{44}^a & \epsilon_2^a\epsilon_3^a + M_{s_{34}}^2 \\ M_{s_{14}}^2 - \mu^a\epsilon_3^a & -B_3^a & \epsilon_1^a\epsilon_3^a + M_{s_{24}}^2 & \epsilon_2^a\epsilon_3^a + M_{s_{34}}^2 & s_{55}^a \end{pmatrix}. \tag{83}$$

The definitions for the parameters:

$$\begin{aligned} s_{11}^a &= \frac{g^2+g'^2}{8}(\upsilon_1^{a^2} - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^{a^2}) + \mu^{a^2} + M_{s_{11}}^2 \\ &= \sum_I \mu^a\epsilon_I^a \frac{\upsilon_{\tilde{\nu}_I}^a}{\upsilon_1^a} + B^a\frac{\upsilon_2}{\upsilon_1^a} - \sum_I M_{s_{1(I+1)}}^2, \\ s_{22}^a &= -\frac{g^2+g'^2}{8}(\upsilon_1^{a^2} - \upsilon_2^2 + \upsilon_{\tilde{\nu}_I}^{a^2}) + \mu^{a^2} + \sum_I \epsilon_I^{a^2} + m_{H^2}^2 \\ &= B^a\frac{\upsilon_1^a}{\upsilon_2} - \sum_I B_I^a \frac{\upsilon_{\tilde{\nu}_I}^a}{\upsilon_2}, \\ s_{(I+2)(I+2)}^a &= \frac{g^2+g'^2}{8}(\upsilon_1^{a^2} - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^{a^2}) + \epsilon_I^{a^2} + M_{s_{(I+1)(I+1)}}^2 \\ &= \mu^a\epsilon_I^a\frac{\upsilon_1^a}{\upsilon_{\tilde{\nu}_e}^a} - B_I^a\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}^a} - \sum_{J\neq I}(\epsilon_I^a\epsilon_J^a + M_{s_{(I+1)(J+1)}}^2)\frac{\upsilon_{\tilde{\nu}_J}^a}{\upsilon_{\tilde{\nu}_I}^a} \quad . \end{aligned} \tag{84}$$

Using Eqs. (61) $\sim$ (67) and Eqs. (69) $\sim$ (76) properly, we find

$$M_{odd}^{a^2} = T_1^{-1} M_{odd}^2 T_1 \tag{85}$$

and the unitary matrix $T_1$ is defined:

$$T_1 = \begin{pmatrix} X_{11}^* & 0 & X_{21}^* & X_{31}^* & X_{41}^* \\ 0 & 1 & 0 & 0 & 0 \\ X_{12}^* & 0 & X_{22}^* & X_{32}^* & X_{42}^* \\ X_{13}^* & 0 & X_{23}^* & X_{33}^* & X_{43}^* \\ X_{14}^* & 0 & X_{24}^* & X_{34}^* & X_{44}^* \end{pmatrix} \quad . \tag{86}$$

It is known from Eq. (85) that $M^{a^2}_{odd}$ and $M^2_{odd}$ have the same eigenvalues and the eigenstates of them are related by $\mathcal{X}^\alpha = T_1 \mathcal{Y}^\alpha$, if $\mathcal{X}^\alpha$ presents an eigenstate of $M^2_{odd}$ and $\mathcal{Y}^\alpha$ presents that of $M^{a^2}_{odd}$ for the same eigenvalue $m^2_\alpha$. For the $CP$-even 'Higgs', in the same way one can find

$$M^{a^2}_{even} = T_1^{-1} M^2_{even} T_1$$

here $T_1$ is the same as that in Eq. (86). So the same conclusion on the 'new' and 'old' relation of the eigenvalues and the eigenstates as that in the $CP$-odd case is obtained.

As for the mass matrix of charged Higgs, the case is more complicated because the right handed sleptons should be included in the mixing . In the basis $\Phi_c = (H_2^{a^{1*}}, H_1^2, \tilde{L}_2^{a^{1*}}, \tilde{L}_2^{a^{2*}}, \tilde{L}_2^{a^{3*}}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$, one find:

$$M^{a^2}_c = T_2^{-1} M^2_c T_2, \tag{87}$$

and the unitary transformation matrix $T_2$ now is

$$T_2 = \begin{pmatrix} X^*_{11} & 0 & X^*_{21} & X^*_{31} & X^*_{41} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ X^*_{12} & 0 & X^*_{22} & X^*_{32} & X^*_{42} & 0 & 0 & 0 \\ X^*_{13} & 0 & X^*_{23} & X^*_{33} & X^*_{43} & 0 & 0 & 0 \\ X^*_{14} & 0 & X^*_{24} & X^*_{34} & X^*_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{88}$$

The same conclusion is obtained as the above neutral Higgs cases.

Now, let us consider the neutral fermions. In the basis $(\Phi^0)^T = (-\lambda_B, -i\lambda^3_A, \psi^1_{H^{a^1}}, \psi^2_{H^2}, \nu_{e^a_L}, \nu_{\mu^a_L}, \nu_{\tau^a_L})$, the mass matrix of neutralino-neutrino is written as

$$\mathcal{M}_N^a = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1^a & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e}^a & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu}^a & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau}^a \\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1^a & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_e}^a & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu}^a & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau}^a \\ -\frac{1}{2}g'\upsilon_1^a & \frac{1}{2}g\upsilon_1^a & 0 & -\frac{1}{2}\mu^a & 0 & 0 & 0 \\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu^a & 0 & \frac{1}{2}\epsilon_1^a & \frac{1}{2}\epsilon_2^a & \frac{1}{2}\epsilon_3^a \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e}^a & \frac{1}{2}g\upsilon_{\tilde{\nu}_e}^a & 0 & \frac{1}{2}\epsilon_1^a & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu}^a & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu}^a & 0 & \frac{1}{2}\epsilon_2^a & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau}^a & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau}^a & 0 & \frac{1}{2}\epsilon_3^a & 0 & 0 & 0 \end{pmatrix}, \tag{89}$$

and we find

$$M_N^a = T_3^{-1} M_N T_3. \tag{90}$$

The $T_3$ unitary matrix is defined:

$$T_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{11}^* & 0 & X_{21}^* & X_{31}^* & X_{41}^* \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & X_{12}^* & 0 & X_{22}^* & X_{32}^* & X_{42}^* \\ 0 & 0 & X_{13}^* & 0 & X_{23}^* & X_{33}^* & X_{43}^* \\ 0 & 0 & X_{14}^* & 0 & X_{24}^* & X_{34}^* & X_{44}^* \end{pmatrix}. \tag{91}$$

Once more the same conclusion on the relations of 'new' and 'old' eigenvalues and eigenstates is reached.

As for the mixing of chargino-charged lepton, the mass matrix from the Lagrangian now can be related as follows:

$$\mathcal{M}_C^a = T_4^{-1} \mathcal{M}_{C_T} I \tag{92}$$

with

$$T_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & X_{11}^* & X_{21}^* & X_{31}^* & X_{41}^* \\ 0 & X_{12}^* & X_{22}^* & X_{32}^* & X_{42}^* \\ 0 & X_{13}^* & X_{23}^* & X_{33}^* & X_{43}^* \\ 0 & X_{14}^* & X_{24}^* & X_{34}^* & X_{44}^* \end{pmatrix} . \tag{93}$$

Since charged fermions are considered here, the situation is a little complicated. We need to make the mass matrix diagonal as the case of SM for quarks i.e. first to diagonalize the mass squared matrix (the combination of the matrix and its conjugate), whereas, owing to the relation Eq. (92), the same conclusion can be obtained too as the above.

Furthermore, it is easy to check the interaction terms are the same no matter to start with what an 'old' Lagrangian or a 'new' Lagrangian: as long as the vertices for the model all turn to represent by means of their eigenvalues (physical value) and corresponding eigenstates (physical states) coordinately, the equivalence for the interactions can be seen clearly. Therefore, the $U(4)$ transformation Eq. (59) indeed is shown a freedom for defining the superfields, and the problem how to fix a model of R-parity violation MSSM emerges. To solve this problem, we would like to suggest two 'conventions' for choices: a) with the freedom to rotate away the VEVs for all sneutrinos; b) with the freedom to rotate away all the bilinear terms of R-parity violation in superpotential. Note that one can apply the freedom only once that is to mean one can make either a) or b) but cannot do both successfully in a general case. In fact, besides the two choices we suggest here, there are many choices to fix the freedom. For instance, one may rotate part of the trilinear lepton-superfield terms or the linear lepton-superfield terms which couple to the quark superfields properly in Eq. (4), for specific convenience.

Now let us show the convention a) first: in fact, Refs. [15,20,21] may be considered as the case. For convenience, let us define the angles $\theta, \phi, \xi$ if all the VEVs are real[4]:

[4]If the VEVs of sneutrinos are not real but complex values i.e. there are spontaneous CP violation

$$
\begin{aligned}
\sin\theta &= \frac{v_{\tilde{\nu}_e}}{\sqrt{v_1^2 + v_{\tilde{\nu}_e}^2}} \;, \\
\sin\phi &= \frac{v_{\tilde{\nu}_\mu}}{\sqrt{v_1^2 + v_{\tilde{\nu}_e}^2 + v_{\tilde{\nu}_\mu}^2}} \;, \\
\sin\xi &= \frac{v_{\tilde{\nu}_\tau}}{\sqrt{v_1^2 + v_{\tilde{\nu}_e}^2 + v_{\tilde{\nu}_\mu}^2 + v_{\tilde{\nu}_\tau}^2}} \;.
\end{aligned}
\tag{94}
$$

Indeed, anyone of the R-parity violation MSSMs with nonzero VEVs of sneutrinos may be rotated to the one where only the Higgs superfield $\hat{H}^1$ has nonzero VEV ($v_1 \neq 0$; $v_{\tilde{\nu}_I} = 0$, with $I = e, \mu, \tau$). The 'rotation matrix' (here the $U(4)$ transformation 'degenerates' just to a rotation) can be decomposed into three rotations as below:

$$
X = R_1 \cdot R_2 \cdot R_3 \;. \tag{95}
$$

Here

$$
R_1 = \begin{pmatrix} \cos\xi & 0 & 0 & \sin\xi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\xi & 0 & 0 & \cos\xi \end{pmatrix} ,
$$

$$
R_2 = \begin{pmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ,
$$

and

$$
R_3 = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .
$$

---

in the model through the VEVs, the discussion here is still valid. Only the change of the discussion is from a rotation into a unitary $U(4)$ transformation.

It is easy further to check that

$$
\begin{aligned}
H_6^0 &= \frac{1}{\upsilon}\Big(\upsilon_d \phi_1^{a^0} - \upsilon_2 \phi_2^0\Big) \\
&= \frac{1}{\upsilon}\Big(\upsilon_1 \phi_1^0 - \upsilon_2 \phi_2^0 + \upsilon_{\tilde{\nu}_e} \phi_{\tilde{\nu}_e}^0 + \upsilon_{\tilde{\nu}_\mu} \phi_{\tilde{\nu}_\mu}^0 + \upsilon_{\tilde{\nu}_\tau} \phi_{\tilde{\nu}_\tau}^0\Big) \ ,
\end{aligned}
\tag{96}
$$

and

$$
\begin{aligned}
H_1^+ &= \frac{1}{\upsilon}\Big(\upsilon_d H_2^{1^a} - \upsilon_2 H_1^{2*}\Big) \\
&= \frac{1}{\upsilon}(\upsilon_1 H_2^{1*} - \upsilon_2 H_1^2 + \upsilon_{\tilde{\nu}_e} \tilde{L}_2^{1*} + \upsilon_{\tilde{\nu}_\mu} \tilde{L}_2^{2*} + \upsilon_{\tilde{\nu}_\tau} \tilde{L}_2^{3*}).
\end{aligned}
\tag{97}
$$

are just the Goldstones for spontaneously breaking the EW gauge symmetry. The rest parts of the models can be checked without difficulty, but to shorten the paper we will not show them here precisely.

The second 'convention' b), which is suggested above, can be 'realized' from anyone of the R-parity violation MSSMs by a proper rotation which is similar to the above, if the coefficients $\epsilon_I$ of the bilinear terms in the superpotential are real, otherwise an according $U(4)$ transformation instead of the rotation to complete the purpose. For convenience in various application let us present the rotation precisely as below:

$$
X' = R_1' \cdot R_2' \cdot R_3' \ . \tag{98}
$$

Here

$$
R_1' = \begin{pmatrix} \cos\xi' & 0 & 0 & \sin\xi' \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\xi' & 0 & 0 & \cos\xi' \end{pmatrix} ,
$$

$$
R_2' = \begin{pmatrix} \cos\phi' & 0 & \sin\phi' & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi' & 0 & \cos\phi' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ,
$$

and

$$R_3' = \begin{pmatrix} \cos\theta' & \sin\theta' & 0 & 0 \\ -\sin\theta' & \cos\theta' & 0 & ) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ,$$

with

$$\sin\theta' = \frac{\mu}{\sqrt{\epsilon_1^2 + \mu^2}} \ , \sin\phi' = \frac{\mu}{\sqrt{\epsilon_1^2 + \epsilon_2^2 + \mu^2}} \ ,$$
$$\sin\xi' = \frac{\mu}{\sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \mu^2}} \ . \tag{99}$$

After the rotation, in superpotential only the term $\mu' \varepsilon_{ij} \hat{H}_i^1 \hat{H}_j^2$ with $\mu' = \sqrt{\mu^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2}$ is survived and the other bilinear terms disappear totally.

Before closing this section, we would like to emphasize again: if one would like to compare different R-parity violation MSSMs and to draw any definite conclusion, he must fix the freedom in defining the four superfields (three leptons and the relevant Higgs which has the same quantum numbers as those of leptons) first, and then carry on the comparisons. Otherwise, the obtained surface 'differences' can be attributed to a different definition on the superfields totally or partly. For convenience in applications and not only to fix the freedom for redefining the superfields, we will further 'simplify' the parameterization in various ways elsewhere [28].

## V. NUMERICAL RESULTS

In this section, to be reference results for further studies, we analyze the masses of neutral Higgs and charginos numerically and show their values in proper ways. We have obtained the mass matrices by setting the three type sneutrinos with non-zero vacuum expectation values and $\epsilon_i \neq 0$ ($i = 1, 2, 3$). However, the matrices are quite big that may obscure the typical features. To simplify the 'problem' and to deduct the parameters, we assume only those terms which related the third generation (only $\tau$-lepton number) of lepton number

is broken, but those to the first two generations are not relevant i.e. the terms relating to the 'first two generation lepton-numbers' disappear correspondly. Furthermore through fixing the freedom for redefining the superfields as discussed in the previous section, for the 'survived' trilinear terms relevant to the third generation leptons in superpotential and SUSY soft breaking terms, we will 'rotate away' them as possible as one can, that only the trilinear terms $\varepsilon_{ij}\lambda_{333}\hat{L}^3_i\hat{L}^3_j\hat{R}^3$ in Eq. (4) and $\varepsilon_{ij}\lambda_{333}\tilde{L}^3_i\tilde{L}^3_j\tilde{R}^3$ in Eq. (5) are kept. Namely in the Section for the numerical calculation, we restrict ourselves to compute the case that the VEV of $\tau$-sneutrino is nonzero, the bilinear terms relevant to $\tau$-family lepton as well as the two trilinear terms $\varepsilon_{ij}\lambda_{333}\hat{L}^3_i\hat{L}^3_j\hat{R}^3$ and $\varepsilon_{ij}\lambda_{333}\tilde{L}^3_i\tilde{L}^3_j\tilde{R}^3$ are present.

Two reasons to make such an assumption that only $\tau$-lepton number is violated:

- Under the assumption, we think the main feature will not be lost too much but the mass matrices will turn much simple.

- According to experimental indications, the $\tau$-neutrino may be the heaviest among the three type neutrinos, and so far the constraints for the $\tau$ lepton rare decays are comparatively loose etc, i.e. the third generation of leptons probably are special.

In the numerical calculations below, input parameters are chosen as: $\alpha = \frac{e^2}{4\pi} = \frac{1}{128}$, $M_Z =$ 91.19GeV, $M_W$ = 80.23GeV, $m_\tau$ = 1.77GeV, but for the parameters $m_1$, $m_2$, we assume $m_1 = m_2 =$ 250GeV and the upper limit on $\tau$-neutrino mass $m_{\nu_\tau} \leq$ 10MeV is taken into account seriously.

Now let us consider the masses of the charginos first, when $\epsilon_1 = \epsilon_2 = 0$, and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, the Eq. (38) becomes:

$$\mathcal{M}_C = \begin{pmatrix} 2m_2 & \frac{e\upsilon_2}{\sqrt{2}S_W} & 0 \\ \frac{e\upsilon_1}{\sqrt{2}S_W} & \mu & \frac{l_3\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{e\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}S_W} & \epsilon_3 & \frac{l_3\upsilon_1}{\sqrt{2}} + \frac{\lambda_{333}\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}} \end{pmatrix}. \tag{100}$$

Because $m^2_\tau$ should be identified as the lightest eigenvalue of the matrix $\mathcal{M}^\dagger_C\mathcal{M}_C$, we should take it as an eigenvalue away first so as not to conflict the measurement of $\tau$ lepton mass. After taking the eigenvalue $m^2_\tau$ away, the survived eigenvalue equation becomes:

$$\lambda^2 - \mathcal{A}_C\lambda + \mathcal{B}_C = 0, \tag{101}$$

and

$$\begin{aligned}
\mathcal{A}_C &= X^2 + Y^2 + 4m_2^2 + \frac{l_3^2\Big(\upsilon_1^2 + \upsilon_{\tilde{\nu}_\tau}^2\Big) + \lambda_{333}^2\upsilon_{\tilde{\nu}_\tau}^2}{2} + \frac{e^2\upsilon^2}{2S_W^2}, \\
\mathcal{B}_C &= \frac{2l_3^2}{m_\tau^2}\Big\{ m_2\upsilon\cos\beta Y\Big(1 + \frac{\lambda_{333}}{l_3}\sin\theta_\upsilon\Big) \\
&\quad + \frac{e^2}{4S_W^2}\upsilon^3\cos^2\beta\sin\beta\Big(\sin^2\theta_\upsilon - \cos^2\theta_\upsilon\Big)\Big\}^2,
\end{aligned} \tag{102}$$

with the parameters $X$, $Y$ are defined by

$$\begin{aligned}
X &= \epsilon_3\cos\theta_\upsilon + \mu\sin\theta_\upsilon, \\
Y &= -\epsilon_3\sin\theta_\upsilon + \mu\cos\theta_\upsilon,
\end{aligned} \tag{103}$$

Therefore the masses of the other two charginos are expressed as:

$$m^2_{\kappa^\pm_{1,2}} = \frac{1}{2}\Big\{\mathcal{A}_C \mp \sqrt{\mathcal{A}_C^2 - 4\mathcal{B}_C}\Big\}. \tag{104}$$

The auxiliary parameter $l_3$ can be fixed by the condition that $Det|m_\tau^2 - \mathcal{M}_c^\dagger\mathcal{M}_c| = 0$. When the values of $m_1$, $m_2$, $\tan\beta$, $\tan\theta_\upsilon$ and Y are fixed, the value of X will be fixed by the mass of $\tau$-neutrino. Trying to take $m_{\nu_\tau} = 0.1$ MeV, we plot the mass of the lightest charginos versus with Y in Fig. 10. The two lines in the figure correspond to $\lambda_{333} = 0$ and $\lambda_{333} = 0.5$ respectively. In the figure, we find that the trilinear effect on the chargino masses is small when $\tan\beta >> 1$ and $\tan\theta_\upsilon < 1$. In Fig. 10(c), the line correspond to $\lambda_{333} = 0.5$ is coincide with the line for $\lambda_{333} = 0.5$. When the $\tan\beta \sim 1$ and $\tan\beta > 1$, the difference between $\lambda_{333} = 0.5$ and $\lambda_{333} = 0$ is large. In the case, $\sqrt{\upsilon_1^2 + \upsilon_{\tilde{\nu}_\tau}^2} \sim \upsilon_2$ and $\upsilon_1 < \upsilon_{\tilde{\nu}_\tau}$ and the effect of $\lambda_{333}\upsilon_{\tilde{\nu}_\tau}$ on the lightest chargino mass cannot be neglected. For comparison and considering the results obtained at Super-K for neutrino oscillations, with a smaller neutrino mass $m_{\nu_\tau} = 10$eV but the same parameters being taken, we do the numerical calculation once more. The obtained curves are different from those in Fig. 10 by certain amount but not qualitatively and we plot them in Fig. 11.

Now, as for the mass-matrices of the neutral Higgs, under the same assumption, the one for CP-even Higgs is truncated to:

$$\mathcal{M}^2_{even} = \begin{pmatrix} r_{11} & -e_{12} - B & e_{15} - \mu\epsilon_3 \\ -e_{12} - B & r_{22} & -e_{25} + B_3 \\ e_{15} - \mu\epsilon_3 & -e_{25} + B_3 & r_{33} \end{pmatrix} \tag{105}$$

with

$$\begin{aligned} r_{11} &= \frac{g^2 + g'^2}{8}(3\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m_{H^1}^2 \\ &= \frac{g^2 + g'^2}{4}\upsilon_1^2 + \Big(\mu\epsilon_3 - m_{HL^3}^2\Big)\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\frac{\upsilon_2}{\upsilon_1}, \\ r_{22} &= \frac{g^2 + g'^2}{8}(-\upsilon_1^2 + 3\upsilon_2^2 - \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m_{H^2}^2 \\ &= \frac{g^2 + g'^2}{4}\upsilon_2^2 + B\frac{\upsilon_1}{\upsilon_2} - B_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}, \\ r_{33} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + 3\upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m_{L^3}^2 \\ &= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\tau}^2 + \Big(\mu\epsilon_3 - m_{HL^3}^2\Big)\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}. \end{aligned} \tag{106}$$

and $e_{12}$, $e_{15}$, $e_{25}$ are defined in Eq. (A2). Whereas the mass matrix of CP-odd Higgs is truncated to:

$$\mathcal{M}^2_{odd} = \begin{pmatrix} s_{11} & B & -\mu\epsilon_3 + m_{HL^3}^2 \\ B & s_{22} & -B_3 \\ -\mu\epsilon_3 + m_{HL^3}^2 & -B_3 & s_{33} \end{pmatrix} \tag{107}$$

with

$$\begin{aligned} s_{11} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m_{H^1}^2 \\ &= \Big(\mu\epsilon_3 - m_{HL^3}^2\Big)\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\frac{\upsilon_2}{\upsilon_1}, \\ s_{22} &= -\frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m_{H^2}^2 \\ &= B\frac{\upsilon_1}{\upsilon_2} - B_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}, \\ s_{33} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m_{L^3}^2 \\ &= \Big(\mu\epsilon_3 - m_{HL^3}^2\Big)\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}. \end{aligned} \tag{108}$$

Introducing the following auxiliary variables:

$$X_s = B,$$
$$Y_s = \mu\epsilon_3 - m^2_{HL^3},$$
$$Z_s = B_3, \tag{109}$$

the masses of the neutral Higgs can be expressed by the parameters $X_s$, $Y_s$, $Z_s$ and $\tan\beta$, $\tan\theta_\upsilon$. For the masses of CP-odd Higgs, the masses of the two CP-odd Higgs are given by

$$m^2_{H^0_{3+2,3}} = \frac{1}{2}(\mathcal{A} \mp \sqrt{\mathcal{A}^2 - 4\mathcal{B}}) \tag{110}$$

if we define:

$$\begin{aligned}\mathcal{A} &= X_s(\frac{\upsilon_1}{\upsilon_2} + \frac{\upsilon_2}{\upsilon_1}) + Y_s(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}) - Z_s(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}),\\ \mathcal{B} &= -Y_s Z_s(\frac{\upsilon_1}{\upsilon_2} + \frac{\upsilon_2}{\upsilon_1}) - X_s Z_s(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}) + X_s Y_s(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}) +\\ &\quad X_s Y_s \frac{\upsilon_1^2}{\upsilon_2 \upsilon_{\tilde{\nu}_\tau}} - X_s Z_s \frac{\upsilon_2^2}{\upsilon_1 \upsilon_{\tilde{\nu}_\tau}} - Y_s Z_s \frac{\upsilon^2_{\tilde{\nu}_\tau}}{\upsilon_1 \upsilon_2}.\end{aligned} \tag{111}$$

In the numerical calculation, we have taken the parameter $\sqrt{|X_s|} = 500$GeV. In Fig. 12, we plot the mass of the lightest CP-even Higgs versus the parameter $\sqrt{|Y_s|}$. The three lines correspond to $\sqrt{|Z_s|} = 60$GeV, 150GeV and 250GeV respectively. From the Fig. 12, we find the mass of the lightest CP-even Higgs turns small when the parameter $\sqrt{|Y_s|}$ turns large. In Fig. 13, we plot the mass of the lightest CP-even Higgs versus the parameter $\sqrt{|Z_s|}$. The three lines correspond to $\sqrt{|Y_s|} = 60$GeV, 300GeV and 400GeV respectively. From the Fig. 13, we find the mass of the lightest Higgs turns large, as the parameter $\sqrt{|Z_s|}$ changes large. From the numerical calculations, we can find certain parameter space that at tree level the lightest Higgs mass can be $m_{H^0_1} \geq 132$GeV, thus for the supersymmetry model without R-parity one cannot obtain such a stringent limit on the lightest Higgs mass as that in the MSSM with R-parity.

As shown above, the results obtained by our numerical and formulation analysis, both confirm the difference from the MSSM with R-parity: the upper bound of the lightest CP-even Higgs mass of the MSSM without R-parity is loosened a lot.

In summary, besides the formal analysis and clarifying the confusion on the freedom for the redefining the fields, with the assumption that only $\tau$-lepton number is broken, we have calculated the mass spectra in the MSSM without R-parity numerically. From the restriction on the neutrino mass: even $m_{\nu_\tau} \leq 10$ eV, we cannot rule out the possibilities with large $\epsilon_3$. The Feynman rules have been derived in the $\prime$t-Hooft Feynman gauge which are convenient when studying the phenomenology beyond tree level of the model. Here, we would like to point out some references have analyzed the $0\nu\beta\beta$-decay in the model [18] and may obtain certain new constraints about the upper limits on the first generation R-parity violating parameters, such as $\epsilon_1$ and $\upsilon_{\tilde{\nu}_e}$; whereas for the other two generations, there are no such serious restrictions on the R-parity violating parameters.

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## APPENDIX A: THE PARAMETERS

### 1. The parameters appearing in the mass matrix for CP-even Higgs

The parameters appearing in the matrix elements are defined as follows:

$$r_{11} = \frac{g^2+g'^2}{8}\Big(3\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\Big) + |\mu|^2 + m_{H^1}^2$$

$$= \frac{g^2+g'^2}{4}\upsilon_1^2 + \sum_I \Big(\mu\epsilon_I - m_{HL^I}^2\Big)\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\frac{\upsilon_2}{\upsilon_1},$$

$$r_{22} = \frac{g^2+g'^2}{8}\Big(-\upsilon_1^2 + 3\upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2\Big) + |\mu|^2 + \sum_I \epsilon_I^2 + m_{H^2}^2$$

$$
\begin{aligned}
&= \frac{g^2+g'^2}{4}\upsilon_2^2 + B\frac{\upsilon_1}{\upsilon_2} - \sum_I B_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\
r_{(I+2)(I+2)} &= \frac{g^2+g'^2}{8}\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_J \upsilon_{\tilde{\nu}_J}^2 + 2\upsilon_{\tilde{\nu}_I}^2\Big) + \epsilon_I^2 + m_{L^I}^2 \\
&= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_I}^2 + \Big(\mu\epsilon_I - m_{HL^I}^2\Big)\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}} - B_I\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}} \\
&- \sum_{J\neq I}\Big(\epsilon_I\epsilon_J + m_{L^{IJ}}^2\Big)\frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}}
\end{aligned} \tag{A1}
$$

and

$$
\begin{aligned}
e_{12} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_2, & e_{13} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_e} + m_{HL^1}^2, \\
e_{14} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\mu} + m_{HL^2}^2, & e_{15} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} + m_{HL^3}^2, \\
e_{23} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_e}, & e_{24} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\mu}, \\
e_{25} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau}, & e_{34} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu} + m_{L^{12}}^2, \\
e_{35} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau} + m_{L^{13}}^2, & e_{45} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau} + m_{L^{23}}^2.
\end{aligned} \tag{A2}
$$

### 2. The parameters appearing in the mass matrix for CP-odd Higgs

The parameters appearing in the matrix elements are defined as follows:

$$
\begin{aligned}
s_{11} &= \frac{g^2+g'^2}{8}\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\Big) + \mu^2 + m_{H^1}^2 \\
&= \sum_I \Big(\mu\epsilon_I + m_{HL^I}^2\Big)\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\frac{\upsilon_2}{\upsilon_1}, \\
s_{22} &= -\frac{g^2+g'^2}{8}\Big(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_I}^2\Big) + \mu^2 + \sum_I \epsilon_I^2 + m_{H^2}^2 \\
&= B\frac{\upsilon_1}{\upsilon_2} - \sum_I B_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\
s_{(I+2)(I+2)} &= \frac{g^2+g'^2}{8}\Big(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\Big) + \epsilon_I^2 + m_{L^I}^2 \\
&= \Big(\mu\epsilon_I - m_{HL^I}^2\Big)\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}} - B_I\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}} - \sum_{J\neq I}\Big(\epsilon_I\epsilon_J + m_{L^{IJ}}^2\Big)\frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}}\ .
\end{aligned} \tag{A3}
$$

### 3. The elements of the charged Higgs mass-matrix

With the interaction basis $\Phi_c = (H_2^{1*},\ H_1^2,\ \tilde{L}_2^{1*},\ \tilde{L}_2^{2*},\ \tilde{L}_2^{3*},\ \tilde{R}^1,\ \tilde{R}^2,\ \tilde{R}^3)$ and Eq. (6), the elements of symmetric mass-matrix $\mathcal{M}_c^2$ for the charged Higgs appearing in Eq. ( 25) may be individually rwitten as follows:

$$
\begin{aligned}
\mathcal{M}^2_{c1,1} &= \frac{g^2}{4}\upsilon_1^2 - \frac{g^2-g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+\mu^2+\sum_I \frac{1}{2}l_I^2\upsilon_{\tilde{\nu}_I}^2+m_{H^1}^2 \\
&= \frac{g^2}{4}(\upsilon_2^2-\sum_I \upsilon_{\tilde{\nu}_I}^2)+\sum_I \frac{1}{2}l_I^2\upsilon_{\tilde{\nu}_I}^2-\sum_I(\mu\epsilon_{\tilde{\nu}_I}-m_{HL^I}^2)\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1}+B\frac{\upsilon_2}{\upsilon_1}, \\
\mathcal{M}^2_{c1,2} &= \frac{g^2}{4}\upsilon_1\upsilon_2+B, \\
\mathcal{M}^2_{c1,3} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_e}-\mu\epsilon_1+m_{HL^1}^2-\frac{1}{2}l_1^2\upsilon_1\upsilon_{\tilde{\nu}_e}-\frac{1}{2}\sum_{IJ}(\lambda_{J1I}-\lambda_{1JI})\upsilon_{\tilde{\nu}_I}\upsilon_{\tilde{\nu}_J}, \\
\mathcal{M}^2_{c1,4} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\mu}-\mu\epsilon_2+m_{HL^2}^2-\frac{1}{2}l_2^2\upsilon_1\upsilon_{\tilde{\nu}_\mu}-\frac{1}{2}\sum_{IJ}(\lambda_{J2I}-\lambda_{2JI})\upsilon_{\tilde{\nu}_I}\upsilon_{\tilde{\nu}_J}, \\
\mathcal{M}^2_{c1,5} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau}-\mu\epsilon_3+m_{HL^3}^2-\frac{1}{2}l_3^2\upsilon_1\upsilon_{\tilde{\nu}_\tau}-\frac{1}{2}\sum_{IJ}(\lambda_{J3I}-\lambda_{3JI})\upsilon_{\tilde{\nu}_I}\upsilon_{\tilde{\nu}_J}, \\
\mathcal{M}^2_{c1,6} &= \frac{1}{\sqrt{2}}l_1\epsilon_1\upsilon_2+l_{s_1}\frac{\upsilon_{\tilde{\nu}_e}}{\sqrt{2}}, \\
\mathcal{M}^2_{c1,7} &= \frac{1}{\sqrt{2}}l_2\epsilon_2\upsilon_2+l_{s_2}\frac{\upsilon_{\tilde{\nu}_\mu}}{\sqrt{2}}, \\
\mathcal{M}^2_{c1,8} &= \frac{1}{\sqrt{2}}l_3\epsilon_3\upsilon_2+l_{s_3}\frac{\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}}, \\
\mathcal{M}^2_{c2,2} &= \frac{g^2}{4}\upsilon_2^2+\frac{1}{8}(g^2-g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+\mu^2+\sum_I \epsilon_I^2+m_{H^2}^2 \\
&= \frac{g^2}{4}(\upsilon_1^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)-\sum_I B_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}+B\frac{\upsilon_1}{\upsilon_2}, \\
\mathcal{M}^2_{c2,3} &= \frac{g^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_e}-B_1, \\
\mathcal{M}^2_{c2,4} &= \frac{g^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\mu}-B_2, \\
\mathcal{M}^2_{c2,5} &= \frac{g^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau}-B_3, \\
\mathcal{M}^2_{c2,6} &= \frac{l_1}{\sqrt{2}}\mu\upsilon_{\tilde{\nu}_e}+\frac{l_1}{\sqrt{2}}\epsilon_1\upsilon_1-\frac{1}{\sqrt{2}}\sum_{IJ}\epsilon_I\lambda_{IJ1}\epsilon_{\tilde{\nu}_J}, \\
\mathcal{M}^2_{c2,7} &= \frac{l_2}{\sqrt{2}}\mu\upsilon_{\tilde{\nu}_\mu}+\frac{l_2}{\sqrt{2}}\epsilon_2\upsilon_1-\frac{1}{\sqrt{2}}\sum_{IJ}\epsilon_I\lambda_{IJ2}\epsilon_{\tilde{\nu}_J}, \\
\mathcal{M}^2_{c2,8} &= \frac{l_3}{\sqrt{2}}\mu\upsilon_{\tilde{\nu}_\tau}+\frac{l_3}{\sqrt{2}}\epsilon_3\upsilon_1-\frac{1}{\sqrt{2}}\sum_{IJ}\epsilon_I\lambda_{IJ3}\epsilon_{\tilde{\nu}_J},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^2_{c3,3} &= \frac{g^2}{4}\upsilon^2_{\tilde{\nu}_e} - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \epsilon_1^2 + \frac{l_1^2}{2}\upsilon_1^2 + m^2_{L^1} \\
&\quad + \sum_J l_1(\lambda_{J11} - \lambda_{1J1})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J1I} - \lambda_{1JI})(\lambda_{M1I} - \lambda_{1MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + (\mu\epsilon_1 - m_{HL^1})\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_e}} - B_1\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_e}} + \frac{l_1^2}{2}\upsilon_1^2 \\
&\quad -(\epsilon_1\epsilon_2 + m^2_{L^{12}})\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}} - (\epsilon_1\epsilon_3 + m^2_{L^{13}})\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}} - \frac{g^2}{4}(\upsilon^2_{\tilde{\nu}_\mu} + \upsilon^2_{\tilde{\nu}_\tau}) \\
&\quad + \sum_J l_1(\lambda_{J11} - \lambda_{1J1})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J1I} - \lambda_{1JI})(\lambda_{M1I} - \lambda_{1MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M}, \\
\mathcal{M}^2_{c3,4} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu} + \epsilon_1\epsilon_2 + \frac{1}{2}\sum_{IJM}(\lambda_{J1I} - \lambda_{1JI})(\lambda_{M2I} - \lambda_{2MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} \\
&\quad + \frac{1}{2}\sum_J l_1(\lambda_{J12} + \lambda_{J21} - \lambda_{1J2} - \lambda_{2J1})\upsilon_1\upsilon_{\tilde{\nu}_J} + m^2_{L^{12}}, \\
\mathcal{M}^2_{c3,5} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau} + \epsilon_1\epsilon_3 + \frac{1}{2}\sum_{IJM}(\lambda_{J1I} - \lambda_{1JI})(\lambda_{M3I} - \lambda_{3MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} \\
&\quad + \frac{1}{2}\sum_J l_1(\lambda_{J13} + \lambda_{J31} - \lambda_{1J3} - \lambda_{3J1})\upsilon_1\upsilon_{\tilde{\nu}_J} + m^2_{L^{13}}, \\
\mathcal{M}^2_{c3,6} &= \frac{1}{\sqrt{2}}l_1\mu\upsilon_2 + \frac{1}{\sqrt{2}}l_{s_1}\upsilon_1 - \frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I11}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I11} - \lambda^s_{1I1})\upsilon_{\tilde{\nu}_I}, \\
\mathcal{M}^2_{c3,7} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I12}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I12} - \lambda^s_{1I2})\upsilon_{\tilde{\nu}_I}, \\
\mathcal{M}^2_{c3,8} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I13}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I13} - \lambda^s_{1I3})\upsilon_{\tilde{\nu}_I}, \\
\mathcal{M}^2_{c4,4} &= \frac{g^2}{4}\upsilon^2_{\tilde{\nu}_\mu} - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \epsilon_2^2 + \frac{l_2^2}{2}\upsilon_1^2 + m^2_{L^2} \\
&\quad + \sum_J l_2(\lambda_{J22} - \lambda_{2J2})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J2I} - \lambda_{2JI})(\lambda_{M2I} - \lambda_{2MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + (\epsilon_2\mu - m^2_{HL^2})\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\mu}} - B_2\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\mu}} + \frac{l_2^2}{2}\upsilon_1^2 \\
&\quad -(\epsilon_1\epsilon_2 + m^2_{L^{12}})\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}} - (\epsilon_2\epsilon_3 + m^2_{L^{23}})\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}} - \frac{g^2}{4}(\upsilon^2_{\tilde{\nu}_e} + \upsilon^2_{\tilde{\nu}_\tau}) \\
&\quad + \sum_J l_2(\lambda_{J22} - \lambda_{2J2})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J2I} - \lambda_{2JI})(\lambda_{M2I} - \lambda_{2MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M}, \\
\mathcal{M}^2_{c4,5} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau} + \frac{1}{2}\sum_{IJM}(\lambda_{J2I} - \lambda_{2JI})(\lambda_{M3I} - \lambda_{3MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} \\
&\quad + \frac{1}{2}\sum_J l_2(\lambda_{J23} + \lambda_{J32} - \lambda_{2J3} - \lambda_{3J2})\upsilon_1\upsilon_{\tilde{\nu}_J} + m^2_{L^{23}},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^2_{c4,6} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I21}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I21}-\lambda^s_{2I1})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c4,7} &= \frac{1}{\sqrt{2}}l_2\mu\upsilon_2 - \frac{1}{\sqrt{2}}l_{s_2}\upsilon_1 - \frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I22}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I22}-\lambda^s_{2I2})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c4,8} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I23}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I23}-\lambda^s_{2I3})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c5,5} &= \frac{g^2}{4}\upsilon^2_{\tilde{\nu}_\tau} - \frac{1}{8}(g^2-g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon^2_{\tilde{\nu}_I}) + \epsilon_3^2 + \frac{l_3^2}{2}\upsilon_1^2 + m^2_{L^3}\\
&\quad + \sum_J l_3(\lambda_{J33}-\lambda_{3J3})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J3I}-\lambda_{3JI})(\lambda_{M3I}-\lambda_{3MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M}\\
&= \frac{g^2}{4}(\upsilon_2^2-\upsilon_1^2) + (\mu\epsilon_3 - m^2_{HL^3})\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\frac{\epsilon_3\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{l_3^2}{2}\upsilon_1^2\\
&\quad -(\epsilon_1\epsilon_3 + m^2_{L^{13}})\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}} - (\epsilon_2\epsilon_3 + m^2_{L^{23}})\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}} - \frac{g^2}{4}(\upsilon^2_{\tilde{\nu}_e}+\upsilon^2_{\tilde{\nu}_\mu})\\
&\quad + \sum_J l_3(\lambda_{J33}-\lambda_{3J3})\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}(\lambda_{J3I}-\lambda_{3JI})(\lambda_{M3I}-\lambda_{3MI})\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M},\\
\mathcal{M}^2_{c5,6} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I31}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I31}-\lambda^s_{3I1})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c5,7} &= -\frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I32}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I32}-\lambda^s_{3I2})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c5,8} &= \frac{1}{\sqrt{2}}l_3\mu\upsilon_{\tilde{\nu}_\tau} - \frac{1}{\sqrt{2}}l_{s_3}\upsilon_1 - \frac{1}{\sqrt{2}}\sum_I \epsilon_I\lambda_{I33}\upsilon_2 + \frac{1}{\sqrt{2}}\sum_I(\lambda^s_{I33}-\lambda^s_{3I3})\upsilon_{\tilde{\nu}_I},\\
\mathcal{M}^2_{c6,6} &= -\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon^2_{\tilde{\nu}_I}) + \frac{1}{2}l_1^2(\upsilon_1^2+\upsilon^2_{\tilde{\nu}_e}) + m^2_{R^1}\\
&\quad -\frac{1}{2}\sum_J l_1\lambda_{1J1}\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}\lambda_{IJ1}\lambda_{IM1}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M},\\
\mathcal{M}^2_{c6,7} &= \frac{1}{2}l_1l_2\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu} - \frac{1}{2}\sum_J l_1\lambda_{1J2}\upsilon_1\upsilon_{\tilde{\nu}_J} - \frac{1}{2}\sum_J l_2\lambda_{2J1}\upsilon_1\upsilon_{\tilde{\nu}_J}\\
&\quad -\frac{1}{2}\sum_{IJM}\lambda_{IJ1}\lambda_{IM2}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} + m^2_{R^{12}},\\
\mathcal{M}^2_{c6,8} &= \frac{1}{2}l_1l_3\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau} - \frac{1}{2}\sum_J l_1\lambda_{1J3}\upsilon_1\upsilon_{\tilde{\nu}_J} - \frac{1}{2}\sum_J l_2\lambda_{3J1}\upsilon_1\upsilon_{\tilde{\nu}_J}\\
&\quad -\frac{1}{2}\sum_{IJM}\lambda_{IJ1}\lambda_{IM3}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M} + m^2_{R^{13}},\\
\mathcal{M}^2_{c7,7} &= -\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon^2_{\tilde{\nu}_I}) + \frac{1}{2}l_1^2(\upsilon_1^2+\upsilon^2_{\tilde{\nu}_\mu}) + m^2_{R^2}\\
&\quad -\frac{1}{2}\sum_J l_2\lambda_{2J2}\upsilon_1\upsilon_{\tilde{\nu}_J} + \frac{1}{2}\sum_{IJM}\lambda_{IJ2}\lambda_{IM2}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M},\\
\mathcal{M}^2_{c7,8} &= \frac{1}{2}l_2l_3\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau} - \frac{1}{2}\sum_J l_1\lambda_{2J3}\upsilon_1\upsilon_{\tilde{\nu}_J} - \frac{1}{2}\sum_J l_2\lambda_{3J2}\upsilon_1\upsilon_{\tilde{\nu}_J}
\end{aligned}
$$

$$-\frac{1}{2}\sum_{IJM}\lambda_{IJ2}\lambda_{IM2}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M}+m^2_{R^{23}},$$

$$\mathcal{M}^2_{c8,8}=-\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon^2_{\tilde{\nu}_I})+\frac{1}{2}l_3^2(\upsilon_1^2+\upsilon^2_{\tilde{\nu}_\tau})+m^2_{R^3}$$

$$-\frac{1}{2}\sum_J l_3\lambda_{3J3}\upsilon_1\upsilon_{\tilde{\nu}_J}+\frac{1}{2}\sum_{IJM}\lambda_{IJ3}\lambda_{IM3}\upsilon_{\tilde{\nu}_J}\upsilon_{\tilde{\nu}_M}. \tag{A4}$$

Note here that to obtain Eq. (A4), Eq. (13) is used sometimes.

## APPENDIX B: THE MIXING OF THE SQUARKS

In the concerning model, the lepton numbers are broken, and due to the VEVs of sneutrinos and trilinear terms the mixing of squarks is affected. In a general case, the matrix of the squarks mixing should be 6×6. Under our assumptions, we do not consider the squarks mixing between different generations. From superpotential Eq. (2) and the soft-breaking terms Eq.(5), we find the up squarks mass matrix of the I-th generation can be written as:

$$\mathcal{M}^2_{U^I}=\begin{pmatrix}\frac{1}{24}(3g^2-g'^2)(\upsilon^2-2\upsilon_2^2)+\frac{u_I^2}{2}\upsilon_2^2+m^2_{Q^I} & \frac{1}{\sqrt{2}}(u_I\mu\upsilon_1-u_I\sum\limits_{J=1}^{3}\epsilon_J\upsilon_{\tilde{\nu}_J}-u_{S_I}\upsilon_2)\\ \frac{1}{\sqrt{2}}(u_I\mu\upsilon_1-u_I\sum\limits_{J=1}^{3}\epsilon_J\upsilon_{\tilde{\nu}_J}-u_{S_I}\upsilon_2) & \frac{1}{6}g'^2(\upsilon^2-2\upsilon_2^2)+\frac{u_I^2}{2}\upsilon_2^2+m^2_{U^I}\end{pmatrix} \tag{B1}$$

where $I=(1,\,2,\,3)$ is the index of the generations. The interaction eigenstates $\tilde{Q}_1^I$ and $\tilde{U}^I$ connect to the two physical (mass) eigenstates $\tilde{U}_I^i$ $(i=1,\,2)$ through

$$\tilde{U}_I^i=Z^{i,1}_{U^I}\tilde{Q}_1^I+Z^{i,2}_{U^I}\tilde{U}^I. \tag{B2}$$

and $Z_{U^I}$ is determined by the condition:

$$Z^\dagger_{U^I}\mathcal{M}^2_{U^I}Z_{U^I}=\mathrm{diag}(M^2_{U_I^1},M^2_{U_I^2}) \tag{B3}$$

In a similar way, we can give the down squarks mass matrix of the I-th generation:

$$\mathcal{M}^2_{D^I}=\begin{pmatrix}-\frac{1}{24}(3g^2+g'^2)(\upsilon^2-2\upsilon_2^2)+\frac{d_I^2}{2}\upsilon_1^2+m^2_{Q^I} & -\frac{1}{\sqrt{2}}(d_I\mu\upsilon_2-d_{S_I}\upsilon_1)\\ -\frac{1}{\sqrt{2}}(d_I\mu\upsilon_2-d_{S_I}\upsilon_1) & -\frac{1}{12}g'^2(\upsilon^2-2\upsilon_2^2)+\frac{d_I^2}{2}\upsilon_1^2+m^2_{D^I}\end{pmatrix} \tag{B4}$$

The fields $\tilde{Q}_2^I$ and $\tilde{D}^I$ relate to the two physical (mass) eigenstates $\tilde{D}_I^i$ $(i=(1,\,2)$:

$$\tilde{D}_I^i=Z^{i,1}_{D^I}\tilde{Q}_2^I+Z^{i,2}_{D^I}\tilde{D}^I,$$

$$Z^\dagger_{D^I}\mathcal{M}^2_{D^I}Z_{D^I}=\mathrm{diag}(M^2_{D_I^1},M^2_{D_I^2}). \tag{B5}$$

## APPENDIX C: THE PRECISE FORMULAS OF $\mathcal{L}_{SSV}$, $\mathcal{L}_{SVV}$ AND $\mathcal{L}_{SSVV}$

### 1. The precise formulas of $\mathcal{L}_{SSV}$

$$\begin{aligned}
\mathcal{L}_{SSV} = & \frac{i}{2}\sqrt{g^2+g'^2}Z_\mu\Big\{\partial^\mu\phi_1^0\chi_1^0 - \phi_1^0\partial^\mu\chi_1^0 - \partial^\mu\phi_2^0\chi_2^0 + \phi_2^0\partial^\mu\chi_2^0 \\
& + \sum_I\Big(\partial^\mu\phi^0_{\tilde{\nu}_I}\chi^0_{\tilde{\nu}_I} - \phi^0_{\tilde{\nu}_I}\partial^\mu\chi^0_{\tilde{\nu}_I}\Big)\Big\} + \frac{1}{2}g\Big\{W_\mu^+\Big[\chi_1^0\partial^\mu H_2^1 - \partial^\mu\chi_1^0 H_2^1 \\
& -\chi_2^0\partial^\mu H_1^{2*} + \partial^\mu\chi_2^0 H_1^{2*} + \sum_I \chi^0_{\tilde{\nu}_I}\partial^\mu\tilde{L}_2^I - \partial^\mu\chi^0_{\tilde{\nu}_I}\tilde{L}_2^I\Big] + h.c.\Big\} \\
& +\frac{i}{2}g\Big\{W_\mu^+\Big[\phi_1^0\partial^\mu H_2^{1*} - \partial^\mu\phi_1^0 H_2^1 + \phi_2^0\partial^\mu H_1^{2*} - \partial^\mu\phi_2^0 H_1^{2*} \\
& +\sum_I(\phi^0_{\tilde{\nu}_I}\partial^\mu\tilde{L}_2^I - \partial^\mu\phi^0_{\tilde{\nu}_I}\tilde{L}_2^I)\Big] - h.c.\Big\} + \Big\{\frac{1}{2}\sqrt{g^2+g'^2}\Big(\cos 2\theta_W Z_\mu \\
& -\sin 2\theta_W A_\mu\Big)\Big[\sum_I\Big(\tilde{L}_2^{I*}\partial^\mu\tilde{L}_2^I - \partial^\mu\tilde{L}_2^{I*}\tilde{L}_2^I\Big) - H_1^{2*}\partial^\mu H_1^2 \\
& +\partial^\mu H_1^{2*}H_1^2 + H_2^{1*}\partial^\mu H_2^1 - \partial^\mu H_2^{1*}H_2^1\Big] + \Big(2\sin^2\theta_W Z_\mu \\
& +2\sin\theta_W\cos\theta_W A_\mu\Big)\Big[\sum_I\Big(\tilde{R}^{I*}\partial^\mu\tilde{R}^I - \partial^\mu\tilde{R}^{I*}\tilde{R}^I\Big)\Big]\Big\} \\
= & \frac{i}{2}\sqrt{g^2+g'^2}C_{eo}^{ij}\Big(\partial^\mu H_{5+i}H_j^0 - H_{5+i}\partial^\mu H_j^0\Big)Z_\mu \\
& +\Big\{\frac{1}{2}gC_{ec}^{ij}\Big(H_i\partial^\mu H_j^- - \partial^\mu H_i H_j^-\Big)W_\mu^+ + h.c.\Big\} \\
& +\Big\{\frac{i}{2}gC_{co}^{ij}\Big(H_{5+i}\partial^\mu H_j^- - \partial^\mu H_{5+i}H_j^-\Big)W_\mu^+ \\
& +h.c.\Big\} + \Big\{\frac{i}{2}\sqrt{g^2+g'^2}\Big[\Big(\cos 2\theta_W\delta^{ij} - C_c^{ij}\Big)Z_\mu\Big(H_i^-\partial^\mu H_j^+ - \partial^\mu H_i^- H_j^+\Big) \\
& -\sin 2\theta_W A_\mu\Big(H_i^-\partial^\mu H_i^+ - \partial^\mu H_i^- H_i^+\Big)\Big]\Big\},
\end{aligned} \tag{C1}$$

with

$$\begin{aligned}
C_{eo}^{ij} &= \sum_{\alpha=1}^{5} Z_{odd}^{i\alpha}Z_{even}^{j\alpha} - 2Z_{odd}^{i2}Z_{even}^{j2}, \\
C_{ec}^{ij} &= \sum_{\alpha=1}^{5} Z_{even}^{i\alpha}Z_{c}^{j\alpha} - 2Z_{even}^{i2}Z_{c}^{j2}, \\
C_{co}^{ij} &= \sum_{\alpha=1}^{5} Z_{odd}^{i\alpha}Z_{c}^{j\alpha} - 2Z_{odd}^{i2}Z_{c}^{j2}, \\
C_{c}^{ij} &= \sum_{\alpha=6}^{8} Z_{c}^{i\alpha}Z_{c}^{j\alpha}.
\end{aligned} \tag{C2}$$

Here the transform matrices $Z_{even}$, $Z_{odd}$ and $Z_c$ are well defined in Sect.II.

### 2. The precise formulas of $\mathcal{L}_{SVV}$

$$\begin{aligned}\mathcal{L}_{SVV} =& \frac{g^2+g'^2}{4}\Big(\upsilon_1\chi_1^0+\upsilon_2\chi_2^0+\sum_I \upsilon_{\tilde{\nu}_I}\chi^0{}_{\tilde{n}u_I}\Big)\Big(Z_\mu Z^\mu + 2\cos^2\theta_W W_\mu^- W_+^\mu\Big)\\ &+\Big\{\frac{g^2+g'^2}{4}\Big[\cos\theta_W\Big(-1+\cos 2\theta_W\Big)Z_\mu W_+^\mu\Big(\upsilon_1 H_2^1-\upsilon_2 H_1^{2*}\\ &+\sum_I \upsilon_{\tilde{\nu}_I}\tilde{L}_2^I\Big)-\cos\theta_W\sin 2\theta_W A_\mu W^{+\mu}\Big(\upsilon_1 H_2^1-\upsilon_2 H_1^{2*}\\ &+\sum_I \upsilon_{\tilde{\nu}_I}\tilde{L}_2^I\Big)\Big]+h.c.\Big\}\\ =& \frac{g^2+g'^2}{4}C_{even}^i\Big(H_i Z_\mu Z^\mu + 2\cos^2\theta_W H_i W_\mu^- W^{+\mu}\Big)\\ &-\frac{g^2+g'^2}{2}S_W C_W\upsilon\Big[S_W Z_\mu W^{+\mu}H_1^- + C_W A_\mu W^{+\mu}H_1^- + h.c.\Big],\end{aligned} \tag{C3}$$

with

$$C_{even}^i = Z_{even}^{i1}\upsilon_1 + Z_{even}^{i2}\upsilon_2 + \sum_I Z_{even}^{iI+2}\upsilon_{\tilde{\nu}_I}\ . \tag{C4}$$

### 3. The precise formulas of $\mathcal{L}_{SSVV}$

The pieces of $\mathcal{L}_{SSVV}$ is given as

$$\begin{aligned}\mathcal{L}_{SSVV} =& -\frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\chi_1^0\chi_1^0+\chi_2^0\chi_2^0+\sum_I\chi_{\tilde{\nu}_I}^0\chi_{\tilde{\nu}_I}^0\Big)Z_\mu Z^\mu\\ &+\cos^2\theta_W\Big(\chi_1^0\chi_1^0+\chi_2^0\chi_2^0+\sum_I\chi_{\tilde{\nu}_I}^0\chi_{\tilde{\nu}_I}^0\Big)W_\mu^- W^{+\mu}\Big]-\frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\phi_1^0\phi_1^0\\ &+\phi_2^0\phi_2^0+\sum_I\phi_{\tilde{\nu}_I}^0\phi_{\tilde{\nu}_I}^0\Big)Z_\mu Z^\mu+\cos^2\theta_W\Big(\phi_1^0\phi_1^0+\phi_2^0\phi_2^0+\phi_{\tilde{\nu}_I}^0\phi_{\tilde{\nu}_I}^0\Big)W_\mu^- W^{+\mu}\Big]\\ &-\frac{g^2+g'^2}{4}\cos\theta_W\Big[\Big(-1+\cos 2\theta_W\Big)Z_\mu W^{+\mu}\Big(\chi_1^0 H_2^1-\chi_2^0 H_1^{2*}\\ &+\sum_I\chi_{\tilde{\nu}_I}^0\tilde{L}_2^I\Big)-\cos\theta_W\sin 2\theta_W A_\mu W^{+\mu}\Big(\chi_1^0 H_2^1-\chi_2^0 H_1^{2*}\\ &+\sum_I\chi_{\tilde{\nu}_I}^0\tilde{L}_2^I\Big)+h.c.\Big]+\frac{i(g^2+g'^2)}{4}\cos\theta_W\Big[\Big(-1+\cos 2\theta_W\Big)Z_\mu W^{+\mu}\Big(\phi_1^0 H_2^1\\ &-\phi_2^0 H_1^{2*}+\sum_I\phi_{\tilde{\nu}_I}^0\tilde{L}_2^I\Big)-\cos\theta_W\sin 2\theta_W A_\mu W^{+\mu}\Big(\phi_1^0 H_2^1\end{aligned}$$

$$
\begin{aligned}
&-\phi_2^0 H_1^{2*} + \sum_I \phi^0_{\tilde{\nu}_I}\tilde{L}_2^I\Big) + h.c.\Big] - \frac{1}{4}(g^2+g'^2)\Big[\sin^2 2\theta_W A_\mu A^\mu \\
&\Big(H_2^{1*}H_2^1 + H_1^{2*}H_1^2 + \sum_I \tilde{L}_2^{I*}\tilde{L}_2^I\Big) \\
&+\cos^2 2\theta_W Z_\mu Z^\mu\Big(H_2^{1*}H_2^1 + H_1^{2*}H_1^2 + \sum_I \tilde{L}_2^{I*}\tilde{L}_2^I\Big) \\
&-\sin 4\theta_W Z_\mu A^\mu\Big(H_2^{1*}H_2^1 + H_1^{2*}H_1^2 + \sum_I \tilde{L}_2^{I*}\tilde{L}_2^I\Big) \\
&+2\cos^2\theta_W\Big(H_2^{1*}H_2^1 + H_1^{2*}H_1^2 + \sum_I \tilde{L}_2^{I*}\tilde{L}_2^I\Big)\Big] \\
&-\sum_I g'^2 \tilde{R}^{I*}\tilde{R}^I B_\mu B^\mu \\
= &-\frac{1}{4}(g^2+g'^2)\Big(\frac{1}{2}H_iH_iZ_\mu Z^\mu + \cos^2\theta_W H_iH_iW_\mu^- W^{+\mu}\Big) \\
&-\frac{1}{4}(g^2+g'^2)\Big(\frac{1}{2}H_{5+i}H_{5+i}Z_\mu Z^\mu + \cos^2\theta_W H_{5+i}H_{5+i}W_\mu^- W^{+\mu}\Big) \\
&+\frac{1}{4}(g^2+g'^2)\sin 2\theta_W\Big\{C_{ec}^{ij}\Big[\sin\theta_W H_i Z_\mu W^{+\mu}H_j^- \\
&+\cos\theta_W H_i A_\mu W^{+\mu}H_j^-\Big] + h.c.\Big\} \\
&-\frac{i}{4}(g^2+g'^2)\sin 2\theta_W\Big\{C_{co}^{ij}\Big[\sin\theta_W H_{5+i} Z_\mu W^{+\mu}H_j^- \\
&+\cos\theta_W H_{5+i} A_\mu W^{+\mu}H_i^-\Big] - h.c.\Big\} \\
&-\frac{1}{4}(g^2+g'^2)\Big\{2\cos^2\theta_W\Big(\delta_{ij}-C_c^{ij}\Big)H_i^-H_j^+W_\mu^-W^{+\mu} \\
&+\Big[\cos^2 2\theta_W\delta_{ij} - C_c^{ij}\Big(4\sin^3\theta_W - \cos^2 2\theta_W\Big)\Big]H_i^-H_j^+Z_\mu Z^\mu \\
&+\sin^2 2\theta_W\delta_{ij}H_i^-H_j^+A_\mu A^\mu + \Big[\sin 4\theta_W\delta_{ij} \\
&-C_c^{ij}\Big(\sin 4\theta_W + 8\sin^2\theta_W\cos\theta_W\Big)\Big]Z_\mu A^\mu H_i^-H_j^+\Big\}\,,
\end{aligned}
\tag{C5}
$$

where the $C_{eo}^{ij}$, $C_{co}^{ij}$ and $C_c^{ij}$ are defined in Eq. (C2).

## APPENDIX D: THE COMPLEMENTARY EXPRESSIONS OF THE COUPLINGS IN $\mathcal{L}_{SSS}$ AND $\mathcal{L}_{SSSS}$

In this appendix, we give precise expressions of the couplings that appear in the $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$. The method has been described clearly in text, the results are:

$$
\begin{aligned}
A_{ec}^{kij} =& \frac{g^2+g'^2}{4}\Big(\upsilon_1 Z_{even}^{k,1}Z_c^{i,1}Z_c^{j,1}+\upsilon_2 Z_{even}^{k,2}Z_c^{i,2}Z_c^{j,2}+\sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,2+I}Z_c^{j,2+I}\Big) \\
&+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{4}+l_I^2\Big)\Big(\upsilon_1 Z_{even}^{k,1}Z_c^{i,2+I}Z_c^{j,2+I}+\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,1}Z_c^{j,1}\Big) \\
&+\sum_{I=1}^{3}\Big(\frac{g^2}{4}-\frac{1}{2}l_I^2\Big)\Big\{\upsilon_1 Z_{even}^{k,2+I}\Big(Z_c^{i,2+I}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,2+I}\Big)+\upsilon_{\tilde{\nu}_I}Z_{even}^{k,1}\Big(Z_c^{i,2+I}Z_c^{j,2} \\
&+Z_c^{i,2}Z_c^{j,2+I}\Big)\Big\}+\frac{g^2-g'^2}{4}\Big(\upsilon_1 Z_{even}^{k,1}Z_c^{i,2}Z_c^{j,2}+\upsilon_2 Z_{even}^{k,2}Z_c^{i,1}Z_c^{j,1} \\
&+\sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,2}Z_c^{j,2}+\upsilon_2 Z_{even}^{k,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big) \\
&+\sum_{I=1}^{3}\Big[\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,5+I}Z_c^{j,5+I}+\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_1 Z_{even}^{k,1}Z_c^{i,5+I}Z_c^{j,5+I} \\
&+\frac{g'^2}{2}\upsilon_2 Z_{even}^{k,2}Z_c^{i,5+I}Z_c^{j,5+I}\Big]+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2}+\upsilon_2 Z_{even}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2} \\
&+Z_c^{i,2}Z_c^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_1 Z_{even}^{k,2}+\upsilon_2 Z_{even}^{k,1}\Big)\Big(Z_c^{i,1}Z_c^{j,2} \\
&+Z_c^{i,2}Z_c^{j,1}\Big)+\frac{1}{\sqrt{2}}l_I\epsilon_3 Z_{even}^{k,1}\Big(Z_c^{i,4}Z_c^{j,2} \\
&+Z_c^{i,2}Z_c^{j,4}\Big)+\frac{1}{\sqrt{2}}\sum_{I=1}^{3}l_I\epsilon_I Z_{even}^{k,2}\Big(Z_c^{i,5+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,5+I}\Big) \\
A_{oc}^{kij} =& \sum_{I=1}^{3}\Big\{\Big(\frac{g^2}{4}-l_I^2\Big)\Big[\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,1}\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big) \\
&+\upsilon_1 Z_{odd}^{k,2+I}\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big)\Big] \\
&+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,2}+\upsilon_2 Z_{odd}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,2} \\
&+\upsilon_2 Z_{odd}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big) \\
&\frac{1}{\sqrt{2}}l_I\epsilon_I Z_{odd}^{k,1}\Big(-Z_c^{i,5+I}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,5+I}\Big) \\
&-\frac{1}{\sqrt{2}}l_I\epsilon_I Z_{odd}^{k,2}\Big(Z_c^{i,5+I}Z_c^{j,1}-Z_c^{i,1}Z_c^{j,5+I}\Big)\Big\} \\
\mathcal{A}_{ec}^{klij} =& \frac{g^2+g'^2}{8s_W c_W}\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,1}Z_c^{j,1}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,2}Z_c^{j,2}+\sum_{I=1}^{3}Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,2+I}Z_c^{j,2+I}\Big) \\
&+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,2+I}Z_c^{j,2+I}+Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,1}Z_c^{j,1}\Big) \\
&+\frac{g'^2-g^2}{8}\Big[Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,2}Z_c^{j,2}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,1}Z_c^{j,1}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{I=1}^{3}\Big(Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,2}Z_c^{j,2}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\Big]\\
&+\frac{g^2}{4}\Big[Z_{even}^{k,1}Z_{even}^{l,2}\Big(Z_c^{i,2}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2}\big)+\sum_{I=1}^{3}Z_{even}^{k,1}Z_{even}^{l,2+I}\big(Z_c^{i,2+I}Z_c^{j,1}\\
&+Z_c^{i,1}Z_c^{j,2+I}\Big)+\sum_{I=1}^{3}Z_{even}^{k,2}Z_{even}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,2+I}\Big)\Big]\\
&-\sum_{I=1}^{3}\Big[\frac{l_I^2}{2}Z_{even}^{k,1}Z_{even}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2+I}\Big)-\frac{g'^2}{4}\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\\
&-Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,5+I}Z_c^{j,5+I}+Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,5+I}Z_c^{j,5+I}\Big)\\
&+\frac{l_I^2}{2}Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{oc}^{klij}=&\frac{g^2+g'^2}{8}\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,1}Z_c^{j,1}+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,2}Z_c^{j,2}\\
&+\sum_{I=1}^{3}Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,2+I}Z_c^{j,2+I}\Big)+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,2+I}Z_c^{j,2+I}\\
&+Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,1}Z_c^{j,1}\Big)+\frac{g'^2-g^2}{8}\Big\{Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,2}Z_c^{j,2}\\
&+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,1}Z_c^{j,1}+\sum_{I=1}^{3}\Big(Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,2}Z_c^{j,2}\\
&+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\Big\}+\frac{g^2}{4}\Big\{Z_{odd}^{k,1}Z_{odd}^{l,2}\Big(Z_c^{i,2}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2}\Big)\\
&+\sum_{I=1}^{3}\Big[Z_{odd}^{k,1}Z_{odd}^{l,3}\Big(Z_c^{i,3}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,3}\Big)+Z_{odd}^{k,2}Z_{odd}^{l,3}\Big(Z_c^{i,3}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,3}\Big)\Big]\Big\}\\
&-\sum_{I=1}^{3}\Big\{\frac{l_I^2}{2}Z_{odd}^{k,1}Z_{odd}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2+I}\Big)-\frac{g'^2}{4}\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\\
&-Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,5+I}Z_c^{j,5+I}+Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,5+I}Z_c^{j,5+I}\Big)\\
&+\frac{l_I^2}{2}Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{eoc}^{klij}=&\sum_{I=1}^{3}\Big[\Big(\frac{g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{even}^{k,2+I}Z_{odd}^{l,1}+Z_{even}^{k,1}Z_{odd}^{l,2+I}\Big)\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big)\\
&-\frac{g^2}{8}\Big(Z_{even}^{k,2}Z_{odd}^{l,2+I}+Z_{even}^{k,2+I}Z_{odd}^{l,2}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big)\Big]\\
&-\frac{g^2}{8}\Big(Z_{even}^{k,1}Z_{odd}^{l,2}+Z_{even}^{k,2}Z_{odd}^{l,1}\Big)\Big(Z_c^{i,1}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2}\Big)
\end{aligned}
$$

$$
\mathcal{A}_{cc}^{ijkl}=\frac{g^2+g'^2}{8}\Big[\sum_{m,n=1}^{5}Z_c^{i,m}Z_c^{j,m}Z_c^{k,n}Z_c^{l,n}+2\sum_{I=1}^{3}Z_c^{i,2+I}Z_c^{j,2+I}\Big(Z_c^{k,1}Z_c^{l,1}-Z_c^{k,2}Z_c^{l,2}\Big)
$$

$$-2Z_c^{i,1}Z_c^{j,1}Z_c^{k,2}Z_c^{l,2}\Big] + \frac{g'^2}{2}\Big[\sum_{I=1}^{3} Z_c^{i,5+I}Z_c^{j,5+I}\Big(-Z_c^{k,5+I}Z_c^{l,5+I} - Z_c^{k,2+I}Z_c^{l,2+I}\Big)$$

$$-\sum_{I=1}^{3} Z_c^{i,5+I}Z_c^{j,5+I}Z_c^{k,1}Z_c^{l,1}\Big] + \sum_{I=1}^{3} l_I^2 Z_c^{i,5+I}Z_c^{j,5+I}Z_c^{k,1}Z_c^{l,1}$$

where the mixing matrices $Z_{even}$, $Z_{odd}$ and $Z_c$ are defined in Eq. (17), Eq. (20) and Eq. (27) respectively.

## APPENDIX E: THE COEFFICIENTS IN THE R-PARITY VIOLATION COUPLINGS OF HIGGS

The precise expressions for the coefficients in the R-parity violation couplings of Higgs: $C_{snn}^{ijm}$, $C_{Lnk}^{ijm}$, $C_{Rnk}^{ijm}$ and $C_{snk}^{ijm}$ are given as the follows:

$$C_{snn}^{ijm} = \Big(\cos\theta_W Z_N^{j2} - \sin\theta_W Z_N^{j1}\Big)\Big(\sum_{\alpha=1}^{5} Z_{even}^{i\alpha} Z_N^{m2+\alpha} - 2Z_{even}^{i2}Z_N^{m4}\Big),$$

$$C_{skk}^{ijm} = \Big(Z_{even}^{i1}Z_+^{j1}Z_-^{m,2} + Z_{even}^{i2}Z_+^{j2}Z_-^{m1} + \sum_{\alpha=3}^{5} Z_{even}^{i\alpha}Z_+^{j1}Z_-^{m\alpha}\Big)$$
$$+\frac{1}{2g}\Big\{\sum_{I=1}^{3} l_I\Big(Z_{even}^{iI+2}Z_+^{jI+2}Z_-^{m2} - Z_{even}^{i1}Z_+^{jI+2}Z_-^{mI+2}\Big)$$
$$+\sum_{IJK}\Big(Z_{even}^{iI+2}Z_+^{jJ+2}Z_-^{mK+2} - Z_{even}^{iK+2}Z_+^{jJ+2}Z_-^{mI+2}\Big)\Big\},$$

$$C_{onn}^{ijm} = \Big(\cos\theta_W Z_N^{j2} - \sin\theta_W Z_N^{j1}\Big)\Big(\sum_{\alpha=1}^{5} Z_{odd}^{i\alpha} Z_N^{m2+\alpha} - 2Z_{odd}^{i2}Z_N^{m4}\Big),$$

$$C_{okk}^{ijm} = \Big(Z_{odd}^{i1}Z_+^{j1}Z_-^{m2} + Z_{odd}^{i2}Z_+^{j2}Z_-^{m1} + \sum_{\alpha=1}^{3} Z_{odd}^{i2+\alpha}Z_+^{j1}Z_-^{m2+\alpha}\Big)$$
$$+\frac{i}{2g}\Big\{\sum_{I} l_I\Big(Z_{odd}^{i2+I}Z_+^{j2+I}Z_-^{m2} - Z_{odd}^{i1}Z_+^{j2+I}Z_-^{m2+I}\Big)$$
$$+\sum_{IJK}\lambda_{IJK}\Big(Z_{odd}^{i2+I}Z_+^{j2+K}Z_-^{mJ+2} - Z_{odd}^{iJ+2}Z_+^{j2+K}Z_-^{m2+I}\Big)\Big\},$$

$$C_{Lnk}^{ijm} = \Big[Z_c^{i1}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_-^{j2}Z_N^{m2} + \sin\theta_W Z_-^{j2}Z_N^{m,1}\Big) - \cos\theta_W Z_-^{j1}Z_N^{m3}\Big)$$
$$+\sum_{\alpha=3}^{5} Z_c^{i\alpha}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_-^{j\alpha}Z_N^{m2} + \sin\theta_W Z_-^{j\alpha}Z_N^{m1}\Big) - \cos\theta_W Z_-^{j1}Z_N^{m2+\alpha}\Big)\Big]$$
$$+\frac{1}{2\sqrt{g^2+g'^2}}\Big\{\sum_{I=1}^{3} l_I\Big(Z_c^{i5+I}Z_-^{j2+I}Z_N^{m3} - Z_c^{i5+I}Z_-^{j2}Z_N^{m4+I}\Big)$$

$$
\begin{aligned}
&+\sum_{IJK}\lambda_{IJK}\Big(Z_c^{i5+I}Z_-^{j2+J}Z_N^{m4+K}-Z_c^{i5+I}Z_-^{j2+K}Z_N^{m4+J}\Big)\Big\},\\
C_{Rnk}^{ijm} &= \Big[Z_c^{i2}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_+^{*j2}Z_N^{*m2}+\sin\theta_W Z_+^{*j2}Z_N^{*m1}\Big)\\
&+\cos\theta_W Z_+^{*j1}Z_N^{*m4}\Big)+\sqrt{2}\sin\theta_W\sum_{I=1}^{3}Z_c^{i5+I}Z_+^{*j2+I}Z_N^{*m1}\Big]\\
&+\frac{1}{2\sqrt{g^2+g'^2}}\Big\{\sum_{I=1}^{3}l_I\Big(Z_c^{i2+I}Z_N^{*m3}Z_+^{*j2+I}-Z_c^{i1}Z_N^{*m3}Z_+^{*j2+I}\Big)\\
&+\sum_{IJK}\lambda_{IJK}\Big(Z_c^{i2+J}Z_N^{*m4+I}Z_+^{*j2+K}-Z_c^{i2+I}Z_N^{*m4+J}Z_+^{*j2+K}\Big)\Big\}\,. \qquad \text{(E1)}
\end{aligned}
$$

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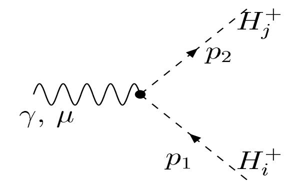
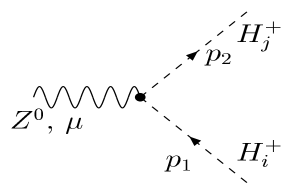
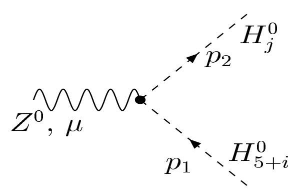


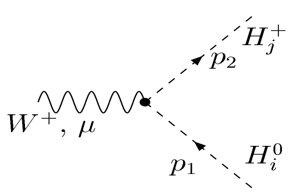

$$-\frac{i}{2}\sqrt{g^2 + g\prime^2}\,\sin 2\theta_W \delta_{ij}(p_1 + p_2)^\mu$$



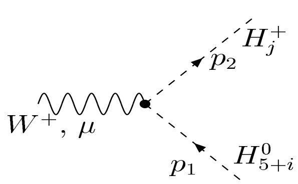

$$\frac{i}{2}\sqrt{g^2 + g\prime^2}\Big(\cos 2\theta_W \delta_{ij} - C_c^{ij}\Big)(p_1 + p_2)^\mu$$




$$\frac{1}{2}\sqrt{g^2 + g\prime^2}C_{eo}^{ij}(p_1 + p_2)^\mu$$

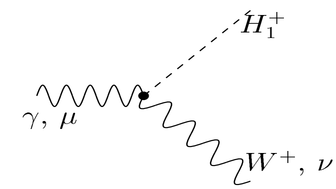



$$\frac{i}{2}\sqrt{g^2 + g\prime^2}\,\cos\theta_W C_{ec}^{ij}(p_1 + p_2)^\mu$$

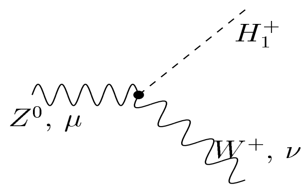



$$\frac{1}{2}\sqrt{g^2 + g\prime^2}\,\cos\theta_W C_{co}^{ij}(p_1 + p_2)^\mu$$

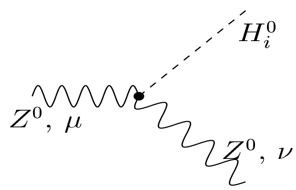
FIG. 1. Feynman rules for SSV vertices, the direction of momentum is indicated above



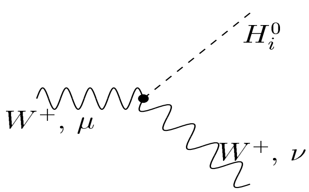

$$-\frac{i}{2}\sqrt{g^2+g\prime^2}\mathrm{M}_W \sin 2\theta_W g_{\mu\nu}$$




$$\frac{i}{2}\sqrt{g^2+g\prime^2}\mathrm{M}_W(\cos 2\theta_W - 1)g_{\mu\nu}$$

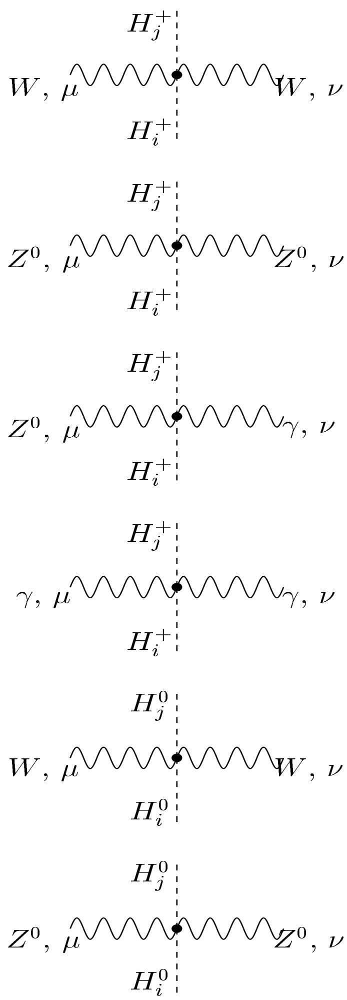



$$-\frac{i}{2}(g^2+g\prime^2)C^i_{even}g_{\mu\nu}$$




$$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W C^i_{even}g_{\mu\nu}$$

FIG. 2. Feynman rules for SVV vertices




$$-\frac{i}{2}(g^2+g'^2)\cos^2\theta_W\Big(\delta_{ij}-C_c^{ij}\Big)g_{\mu\nu}$$

$$-\frac{i}{2}(g^2+g'^2)\Big(\cos^2 2\theta_W\delta_{ij}-C_c^{ij}\Big(4\sin^3\theta_W-\cos^2 2\theta_W\Big)\Big)g_{\mu\nu}$$

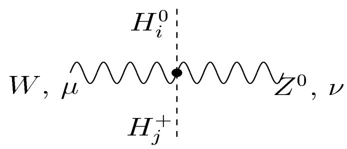
$$-\frac{i}{4}(g^2+g'^2)\Big[\sin 4\theta_W\delta_{ij}-C_c^{ij}\Big(\sin 4\theta_W+8\sin^2\theta_W\cos\theta_W\Big)\Big]g_{\mu\nu}$$

$$-\frac{i}{2}(g^2+g'^2)\sin^2 2\theta_W\delta_{ij}g_{\mu\nu}$$

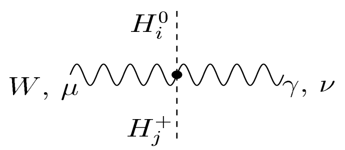
$$-\frac{i}{2}(g^2+g'^2)\cos^2\theta_W\delta_{ij}g_{\mu\nu}$$

$$-\frac{i}{2}(g^2+g'^2)\delta_{ij}g_{\mu\nu}$$

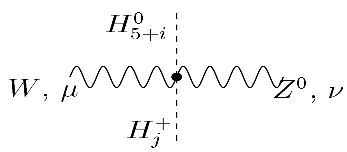
FIG. 3. Feynman rules for SSVV vertices. Part(I)



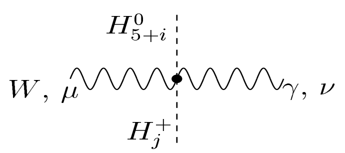

$\frac{i}{2}(g^2 + g'^2)\cos\theta_W \sin^2\theta_W C_{ec}^{ij} g_{\mu\nu}$



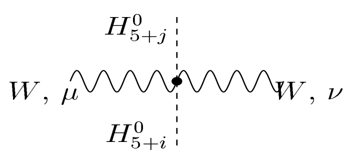

$-\frac{i}{2}(g^2 + g'^2)\cos^2\theta_W \sin\theta_W C_{ec}^{ij} g_{\mu\nu}$



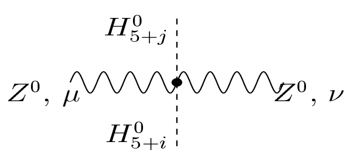

$-\frac{1}{2}(g^2 + g'^2)\cos\theta_W \sin^2\theta_W C_{co}^{ij} g_{\mu\nu}$




$\frac{1}{2}(g^2 + g'^2)\cos^2\theta_W \sin\theta_W C_{co}^{ij} g_{\mu\nu}$

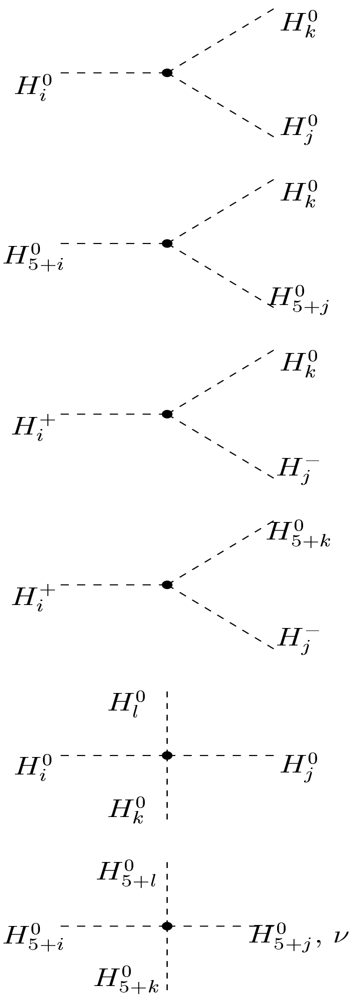



$-\frac{i}{2}(g^2 + g'^2)\cos^2\theta_W \delta_{ij} g_{\mu\nu}$




$-\frac{i}{2}(g^2 + g'^2)\delta_{ij} g_{\mu\nu}$

FIG. 4. Feynman rules for SSVV vertices. Part(II)




$$-\frac{i}{8}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{B}^{k}_{even}$$

$$-\frac{i}{8}(g^2 + g\prime^2)\mathcal{A}^{ij}_{odd}\mathcal{B}^{k}_{even}$$

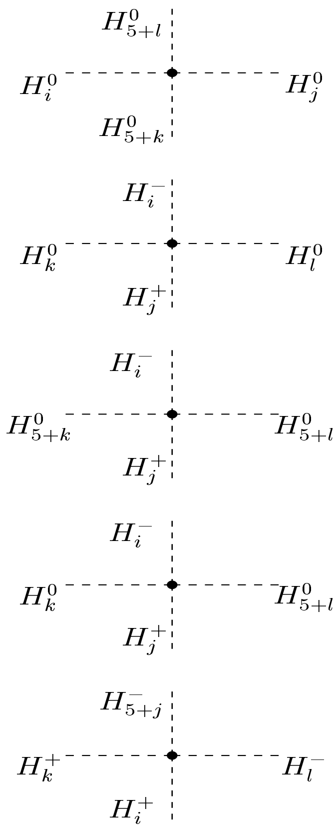
$$-i\mathcal{A}^{kij}_{ec}$$

$$-\mathcal{A}^{kij}_{oc}$$

$$-\frac{i}{32}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{A}^{ij}_{even}$$

$$-\frac{i}{32}(g^2 + g\prime^2)\mathcal{A}^{ij}_{odd}\mathcal{A}^{ij}_{odd}$$

FIG. 5. Feynman rules for the self-coupling of Higgs. Part(I)




$$\frac{i}{16}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{A}^{kl}_{odd}$$

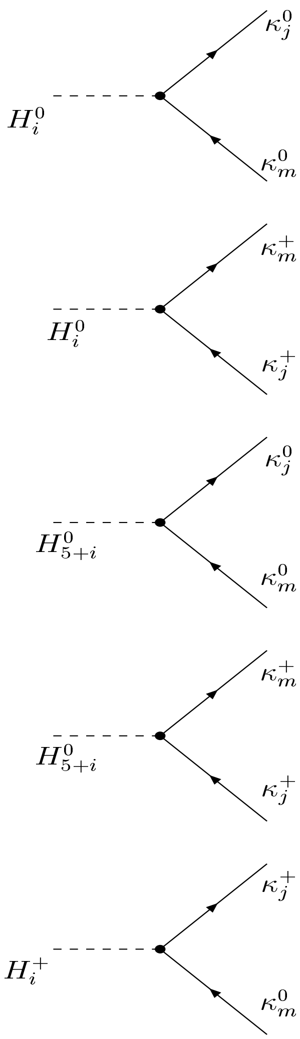
$$-i\mathcal{A}^{klij}_{ec}$$

$$-i\mathcal{A}^{klij}_{oc}$$

$$\mathcal{A}^{klij}_{eoc}$$

$$-i\mathcal{A}^{klij}_{cc}$$

FIG. 6. Feynman rules for the self-coupling of Higgs. Part(II)




$$i\frac{\sqrt{g^2+g\prime^2}}{2}\left(C_{snn}^{jm}P_L + C_{snn}^{jm*}P_R\right)$$

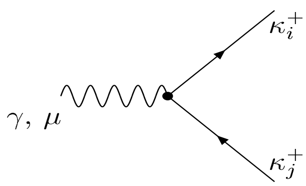
$$i\frac{g}{\sqrt{2}}\left(C_{skk}^{jm}P_L + C_{skk}^{jm*}P_R\right)$$

$$-\frac{\sqrt{g^2+g\prime^2}}{2}\left(C_{onn}^{jm}P_L - C_{onn}^{jm*}P_R\right)$$

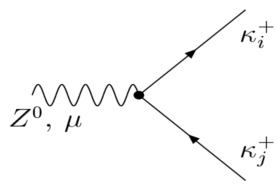
$$-\frac{g}{\sqrt{2}}\left(C_{okk}^{jm}P_L - C_{okk}^{jm*}P_R\right)$$

$$i\sqrt{g^2+g\prime^2}\left(C_{Lnk}^{ij}P_L - C_{Rnk}^{ij}P_R\right)$$

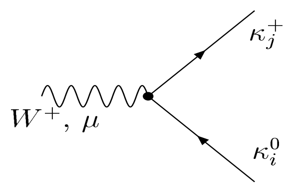
FIG. 7. Feynman rules for the coupling of Higgs with charginos or neutralinos.



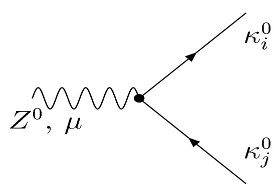

$$i\sqrt{g^2+g\prime^2}\cos\theta_W\sin\theta_W\gamma^\mu\delta_{ij}$$




$$i\sqrt{g^2+g\prime^2}\gamma^\mu[\cos^2\theta_W\delta_{ij}+(\tfrac{1}{2}Z_+^{*i,2}Z_+^{j,2}-\sum_{I=1}^{3}Z_+^{*i,2+I}Z_+^{j,2+I})P_L$$
$$+\tfrac{1}{2}(Z_-^{*i,2}Z_-^{j,2}+\sum_{I=1}^{3}Z_-^{*i,2+I}Z_-^{j,2+I})P_R]$$

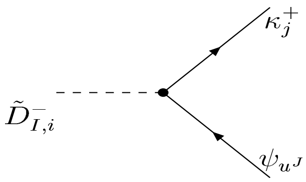



$$ig\gamma^\mu[(-Z_+^{*i,1}Z_N^{j,2}+\tfrac{1}{\sqrt{2}}Z_+^{*i,2}Z_N^{j,4})P_L+(Z_N^{*i,2}Z_-^{j,1}$$
$$+\tfrac{1}{\sqrt{2}}(Z_N^{*i,3}Z_-^{j,2}+\sum_{I=1}^{3}Z_N^{*i,4+I}Z_-^{j,2+I}))P_R]$$

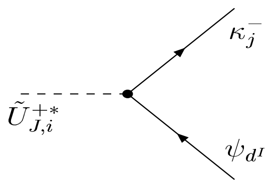



$$i\frac{\sqrt{g^2+g\prime^2}}{2}\Big[\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4}-\big(Z_N^{*i,3}Z_N^{j,3}+\textstyle\sum_{\alpha=5}^{7}Z_N^{*i,\alpha}Z_N^{j,\alpha}\big)\Big)P_L$$
$$-\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4}-\big(Z_N^{*i,3}Z_N^{j,3}+\textstyle\sum_{\alpha=5}^{7}Z_N^{*i,\alpha}Z_N^{j,\alpha}\big)\Big)P_R\Big]$$

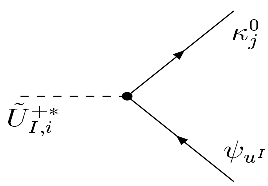
FIG. 8. Feynman rules for the coupling of gauge bosons with charginos or neutralinos.



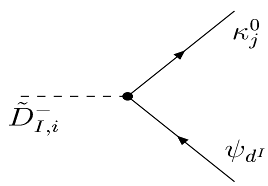

$$iC^{IJ}\Big\{\Big(-gZ^{i,1}_{D^I}Z^{j,1}_{-}+\tfrac{d^I}{2}Z^{i,2}_{D^I}Z^{j,2}_{-}+\tfrac{\lambda'_{KIJ}}{2}Z^{i,2}_{D^I}Z^{j,2+K}_{-}\Big)P_L + \tfrac{u^J}{2}Z^{j,2*}_{+}Z^{i,1}_{D^I}P_R\Big\}$$




$$iC^{IJ*}\Big\{\Big(-gZ^{i,1}_{U^J}Z^{j,1}_{+}+\tfrac{u^J}{2}Z^{i,2}_{U^J}Z^{j,2}_{+}\Big)P_L - \Big(\tfrac{d^I}{2}Z^{j,2*}_{-}Z^{i,1*}_{U^J}+\tfrac{\lambda'_{KIJ}}{2}Z^{j,2+K*}_{-}Z^{i,1*}_{U^J}\Big)P_R\Big\}$$

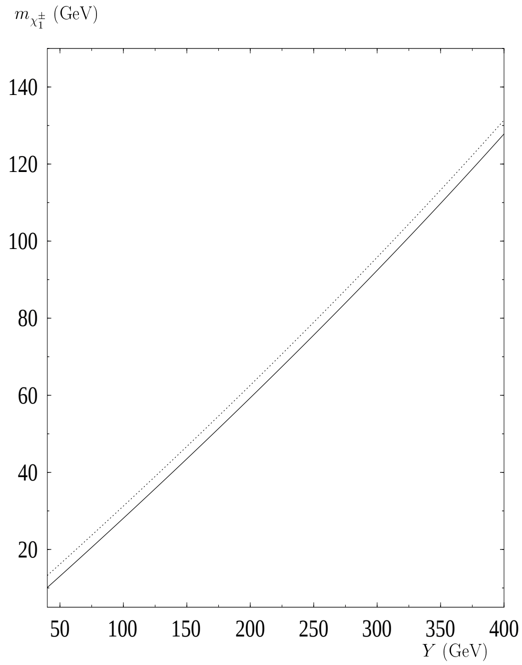



$$i\Big\{\Big[\tfrac{\sqrt{g^2+g'^2}}{\sqrt{2}}Z^{i,1*}_{U^I}\Big(\cos\theta_W Z^{i,2}_N+\tfrac{1}{3}\sin\theta_W Z^{j,1}_N\Big)-\tfrac{u^I}{2}Z^{i,1*}_{U^I}Z^{j,4}_N\Big]P_L + \Big[\tfrac{2\sqrt{2}}{3}g'Z^{i,2*}_{U^I}Z^{j,1}_N-\tfrac{u^I}{2}Z^{i,1*}_{U^I}Z^{j,4*}_N\Big]P_R\Big\}$$

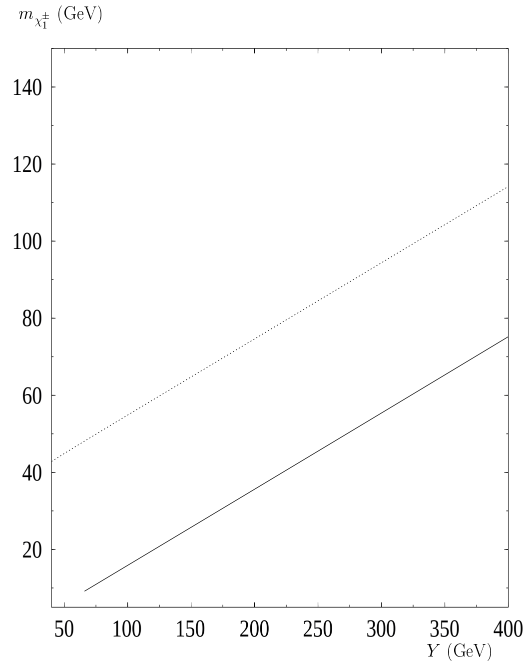



$$i\Big\{\Big[\tfrac{\sqrt{g^2+g'^2}}{\sqrt{2}}Z^{i,1*}_{D^I}\Big(-\cos\theta_W Z^{i,2}_N+\tfrac{1}{3}\sin\theta_W Z^{j,1}_N\Big)+\tfrac{d^I}{2}Z^{i,2}_{D^I}Z^{j,3}_N + \tfrac{\lambda'_{KII}}{2}Z^{i,2}_{D^I}Z^{j,4+K}_N\Big]P_L + \Big[-\tfrac{\sqrt{2}}{3}g'Z^{i,2*}_{D^I}Z^{j,1}_N+\tfrac{d^I}{2}Z^{i,1*}_{D^I}Z^{j,3*}_N+\tfrac{\lambda'_{KII}}{2}Z^{i,1*}_{D^I}Z^{j,4+K*}_N\Big]P_R\Big\}$$

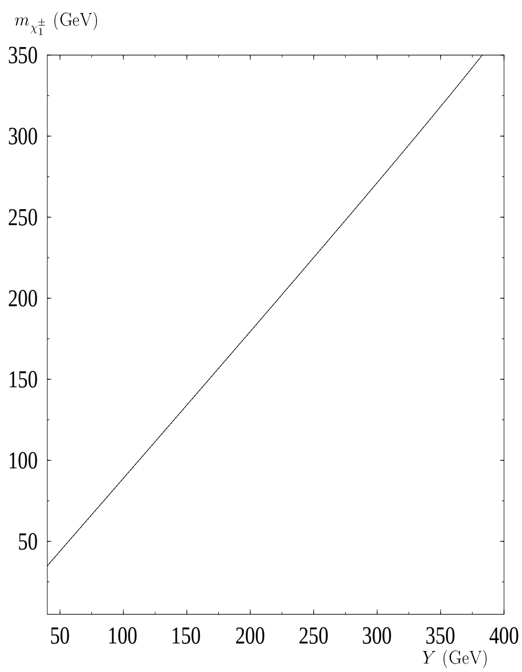
FIG. 9. Feynman rules for the coupling of quarks, squarks with charginos or neutralinos.



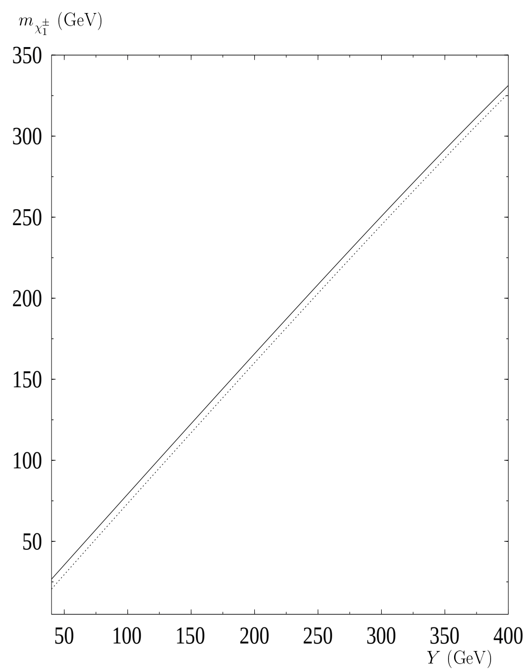

(a)




(b)

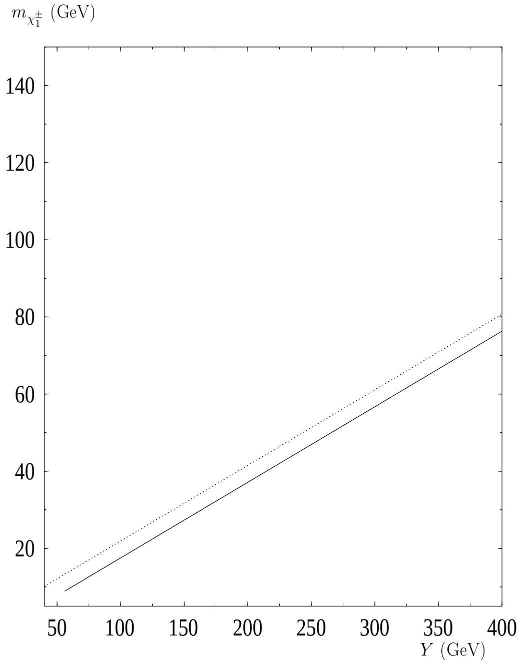



(c)

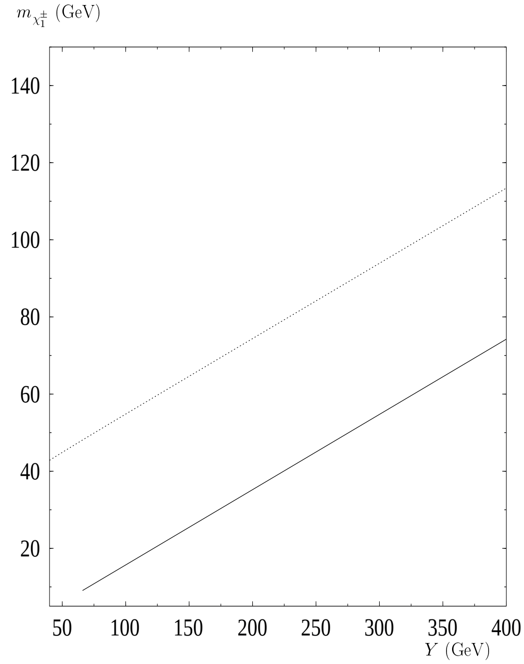



(d)

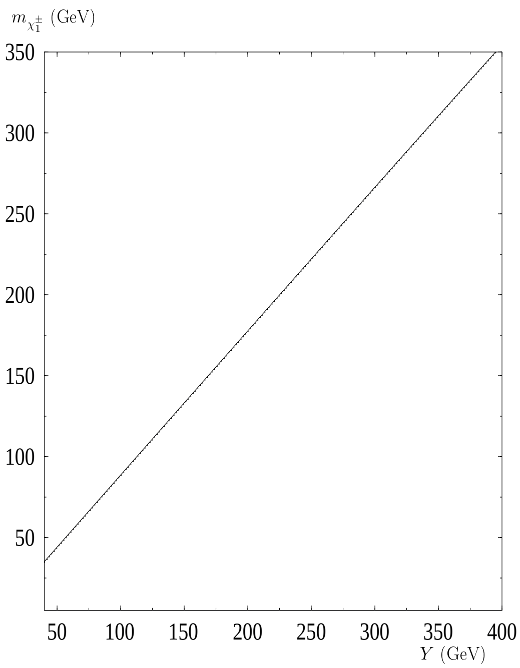
FIG. 10. The mass of the lightest chargino versus Y. The parameters are assigned as $m_1 = m_2 = 250$GeV, $m_{\nu_\tau} = 0.1$MeV and (a)$\tan\beta = 20$, $\tan\theta_v = 5$; (b)$\tan\beta = 2$, $\tan\theta_v = 5$; (c)$\tan\beta = 20$, $\tan\theta_v = 0.5$; (d)$\tan\beta = 2$, $\tan\theta_v = 0.5$. The dot lines correspond to $\lambda_{333} = 0.5$, the solid lines correspond to $\lambda_{333} = 0$.



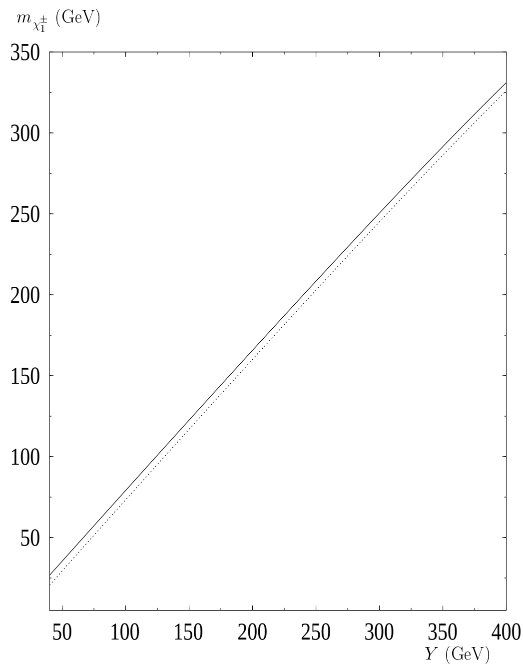

(a)




(b)




(c)




(d)

FIG. 11. The mass of the lightest chargino versus Y. The parameters are assigned as $m_1 = m_2 = 250$GeV, $m_{\nu_\tau} = 0.01$MeV and (a)$\tan\beta = 20$, $\tan\theta_\upsilon = 5$; (b)$\tan\beta = 2$, $\tan\theta_\upsilon = 5$; (c)$\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; (d)$\tan\beta = 2$, $\tan\theta_\upsilon = 0.5$. The dot lines correspond to $\lambda_{333} = 0.5$, the solid lines correspond to $\lambda_{333} = 0$.

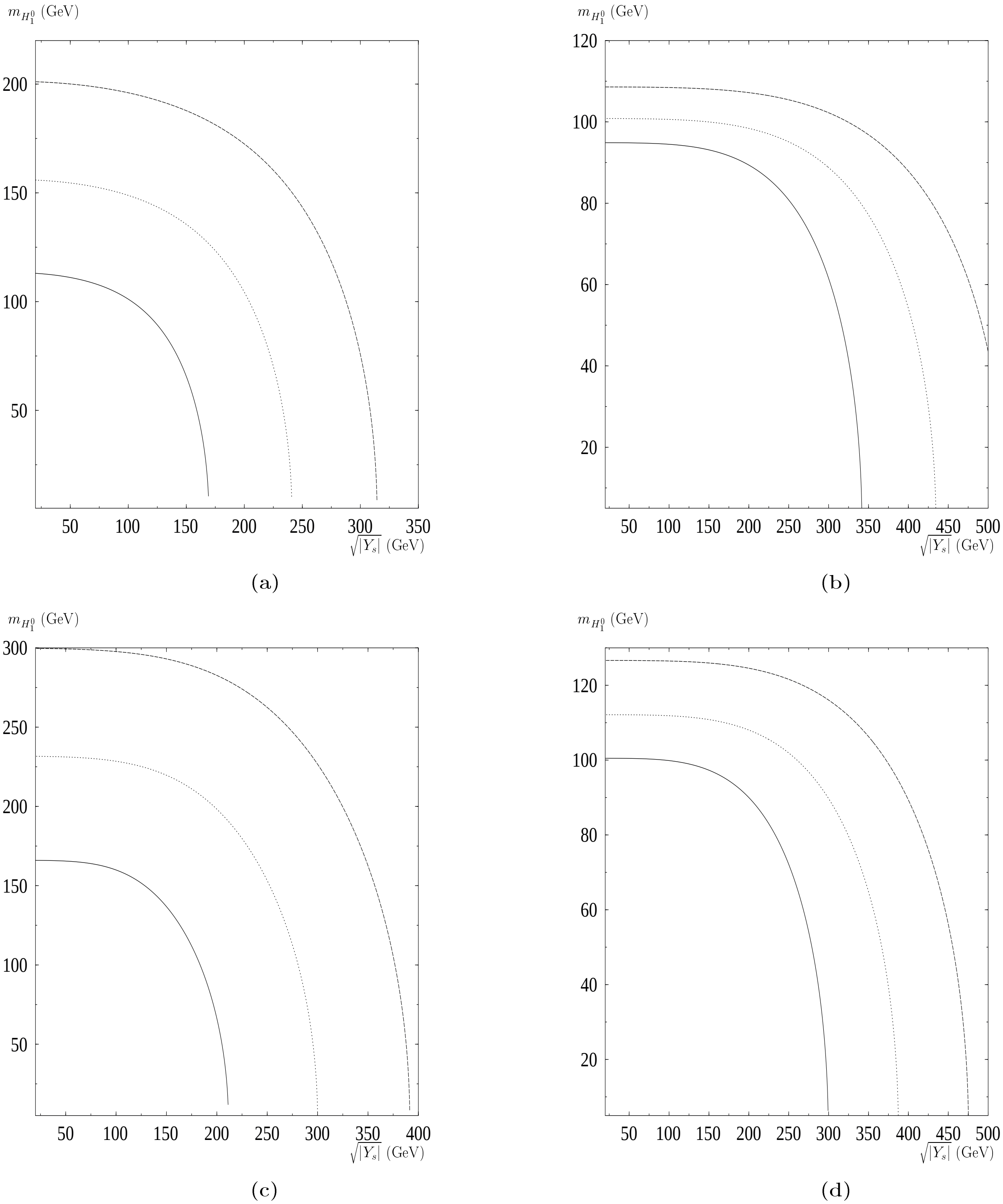



FIG. 12. The mass of the lightest CP-even Higgs versus $\sqrt{|Y_s|}$. The parameters are assigned as $\sqrt{|X_s|} = 500$GeV and (a)$\tan\beta = 20$, $\tan\theta_\upsilon = 5$; (b)$\tan\beta = 2$, $\tan\theta_\upsilon = 5$; (c)$\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; (d)$\tan\beta = 2$, $\tan\theta_\upsilon = 0.5$. The dot-dash lines correspond to $\sqrt{|Z_s|} = 250$GeV, the dash lines correspond to $\sqrt{|Z_s|} = 150$GeV, and dot lines correspond to $\sqrt{|Z_s|} = 60$GeV

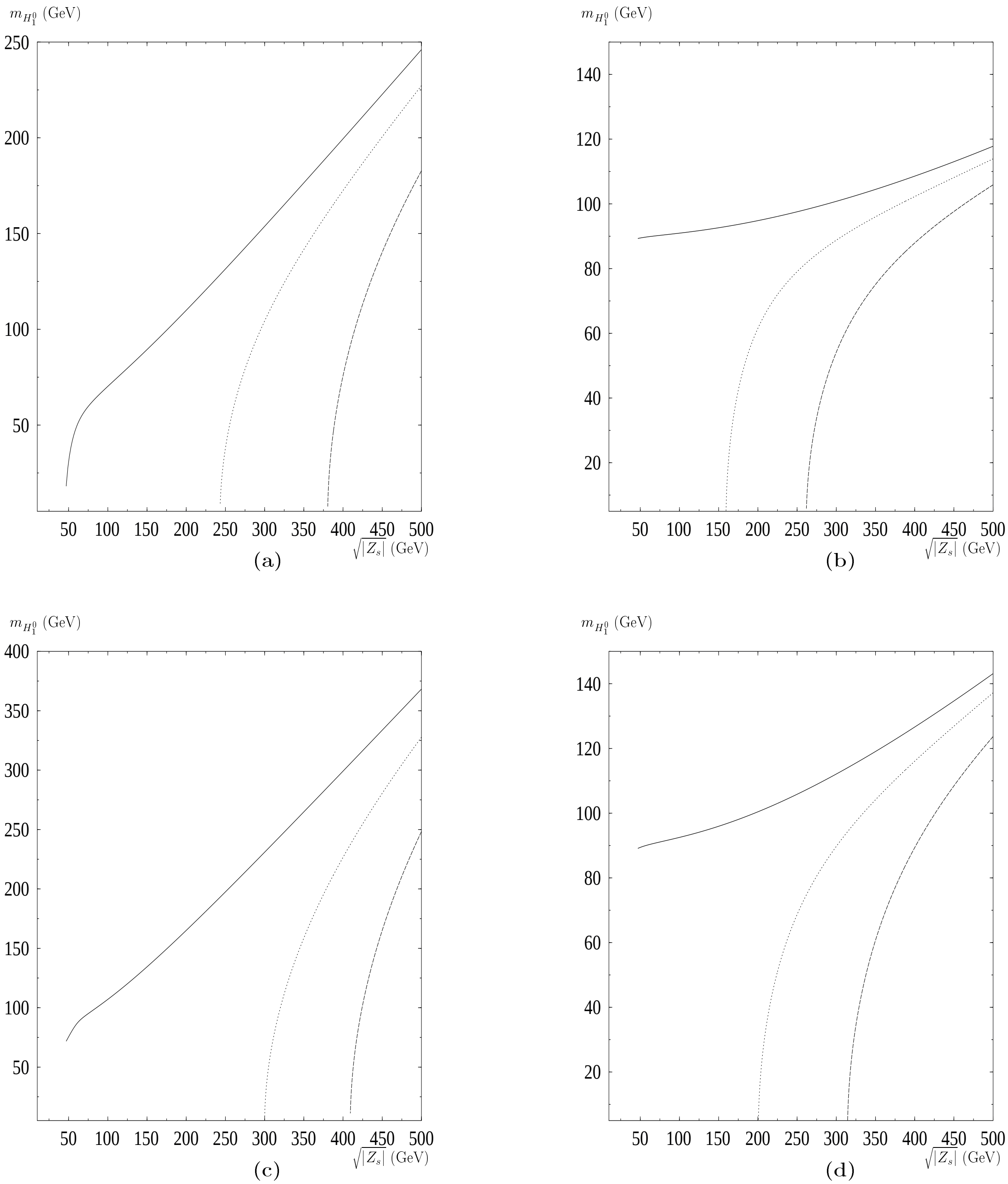



FIG. 13. The mass of the lightest CP-even Higgs versus $\sqrt{|Z_s|}$. The parameters are assigned as $\sqrt{|X_s|} = 500$GeV and (a)$\tan\beta = 20$, $\tan\theta_\upsilon = 5$; (b)$\tan\beta = 2$, $\tan\theta_\upsilon = 5$; (c)$\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; (d)$\tan\beta = 2$, $\tan\theta_\upsilon = 0.5$. The dot-dash lines correspond to $\sqrt{|Y_s|} = 400$GeV, the dash lines correspond to $\sqrt{|Y_s|} = 300$GeV, and dot lines correspond to $\sqrt{|Y_s|} = 60$GeV